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SELF-CONSISTENT THEORY OF HADRON-NUCLEUS SCATTERING. APPLICATION TO PION PHYSICS

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Summary

The requirement of using self-consistent amplitudes to evaluate microscopically the scattering of strongly interacting particles from nuclei is developed. Application of the idea to a simple model of pion-nucleus scattering is made. Numerical results indicate that the expansion of the optical potential converges when evaluated in terms of fully self-consistent quantities. A comparison of the results to a recent determination of the spreading interaction in the phenomenological isobar-hole model shows that the theory accounts for the sign and magnitude of the real and imaginary part of the spreading interaction with no adjusted parameters. The self-consistent theory has a strong density-dependence and the consequences of this for pion-nucleus scattering are discussed.

I. Introduction

The first part of this set of two seminars will consist of a review of several of the important accomplishments made in the last few years in the field of pion-nucleus physics. This is intended as an introduction, for those who may be unfamiliar with the field of pion physics, to concepts which will be used throughout the development of the idea of self-consistency. Next I will discuss some questions raised by these accomplishments and show that for some very natural reasons the commonly employed theoretical methods can not be applied to answer these questions. This situation leads to the idea of self-consistency, which is first explained in a general context. The remainder of the seminars are devoted to illustrating the idea within a simple multiple scattering model for the case of pion scattering. Numerical results for this

application will be presented. An evaluation of the effectiveness of the self-consistent requirement to produce a solution to the model will be made, and a few of the questions raised by recent accomplishments in the field of pion physics will be addressed in the model. Finally, the results of the model calculation will be compared to experimental data and implications of the results discussed.

The field of pion-nucleus physics is a relatively new field and has become quite active only in the last decade. In contrast to, say, the development of proton-nucleus theory, answers to questions of a microscopic nature have been sought early in the development of the field. One might say that the most interesting question addressed, and the one which has been studied with most intense effort, is: "to what extent can pion-nucleus scattering be understood as a succession of elementary, free, pion-nucleon scatterings?" The answer is that, at least in the energy region of 75 to 300 MeV incident pion kinetic energy, the isobar Δ_{33} dominates the scattering, (the Δ_{33} is a pion-nucleon resonance with spin and isospin 3/2 and relative pion-nucleon angular momentum $\ell = 1$) but that the position and width of the isobar are strongly modified by the influence of the nuclear medium.

This result has been most convincingly established within the "isobar-hole" phenomenology,[1,2] which is discussed in detail in Professor Feshbach's lectures.[3] Essentially, the isobar-hole model[1-5] is a coupled channel approach which allows the isobar degrees of freedom to be isolated and treated explicitly. The special features of this approach are the following:

(1) Isobar and nucleon binding and recoil are carefully treated. This is a very important feature of the model. Although the physics here may not seem very exciting, it has proved to be a very difficult technical and apparently numerically significant accomplishment.

(2) Pauli suppression is evaluated. Pauli suppression refers to the requirement that Δ_{33} intermediate states should not consist of a nucleon which has scattered into one of the normally occupied nuclear states.

(3) A phenomenological isobar "shell-model" potential is incorporated.[2] It has the form

$$W(r) = W_0 \rho(r)/\rho_0 + \vec{L} \cdot \vec{S}_\Delta \, W^\Delta_{LS}(r) \qquad (1.1)$$

The (complex) constants W_0 and W_{LS} are adjusted to fit the elastic scattering data.

The most recent results of the isobar-hole model[2] are that the strength of the spin-orbit potential is comparable in size to the nucleon-nucleus spin-orbit force and that W_0 is constant in energy from about 75 MeV to 250 MeV with values $\mathrm{Re}W_0 = 30$ MeV and $\mathrm{Im}W_0 = -40$ MeV.

The application of the isobar-hole model requires a very large computational effort and for practical reasons of computer core size and time limitations the model can not be applied to nuclei with $A \geq 16$. The only technically feasible approach for heavy nuclei is that of the optical model. The optical potential represents in an average way the effect of the nuclear medium on the projectile. There are several microscopic theories of the optical potential which permit its evaluation systematically from an underlying Hamiltonian or Lagrangian formulation. One such approach is the Watson or KMT multiple scattering formalism which is reviewed in Professor Feshbach's lectures.[3] These formalisms are not fully satisfactory for the scattering of pions due to the fact that in nature number of pions is not conserved, but nevertheless the approach is often used in pion physics. Another systematic approach uses diagrammatic perturbation theory and the Dyson equation[6] to calculate the amplitude. This approach does apply to the case of the pion, and in this theory the optical potential is the pion proper self energy. Unfortunately, the language is similar in these two approaches but in some instances the meaning of the language is not exactly the same, which naturally leads to communication difficulties. I shall use the Dyson equation and Feynman diagrams in these lectures.

In any case, if the interaction between the pion and the nucleon is assumed to be a potential, either of the two approaches will, by its own

route, lead to the same optical potential U. In practice one often expands the optical potential as

$$U = \sum_{n=1} U_n \tag{1.2}$$

where U_n has n factors of the density. The leading term U_1 is in principle uniquely related to the (off shell) free pion-nucleon scattering amplitude and the nuclear density matrix. The higher order terms depend in a much more complicated way on the details of the nuclear and scattering dynamics.

The most extensive analysis of pion elastic scattering in an optical model approach has been carried out by Stricker, McManus and Carr[7] and by Liu and Shakin.[8] In these works U_1 is fixed by the relationship to the free pion-nucleon amplitude just mentioned. The term U_2 is then, essentially, adjusted to reproduce the elastic scattering data. The main result for the purposes of this lecture is that a substantial U_2 correction was found.

We have therefore the same conclusion from both the isobar-hole phenomenology and from the optical model phenomenology, namely that there are substantial corrections to the simple picture of pion-nucleon scattering which would describe the scattering as a succession of elementary, free, scatterings of the pion from the constituents of the nucleus. The quantities W_0 and U_2 are different (but essentially equivalent) measures of the correction.

II. Open Questions

The question of fundamental concern is now to understand the origin of the correction terms. That they are somehow related to the true absorption (a reaction process in which a pion is present in the initial but not the final state) is universally believed, but the details of this relationship could be rather subtle. It will take years of careful theoretical and experimental work to understand this process in sufficient detail.

The true absorption of the pion is just one aspect of understanding the correction terms and may actually not be the most interesting aspect. The theory develops some interesting difficulties even in the absence of the true absorption of the pion. This arises from the fact that the pion-nucleon interaction is very strong, as explained below, from threshold to beyond the position of the Δ_{33} resonance.

There are two reasons why the pion-nucleon interaction must be regarded as strong. The first is that the interaction has a very strong off-shell dependence. We write the dominant P-wave amplitude in a separable form, letting $\underset{\sim}{t}$ be the off-shell pion momentum, as

$$\langle \underset{\sim}{t}|f_\omega|\underset{\sim}{t}'\rangle = \frac{v(t)}{v(k)}\, f(0)\, \frac{v(t')}{v(k)}\, \hat{\underset{\sim}{t}} \cdot \hat{\underset{\sim}{t}}' \qquad (2.1)$$

where f(0) is the on-shell, forward scattering amplitude (which is therefore ω-dependent) k is the on-shell momentum ($\omega^2 = k^2 + m_\pi^2$), and v(t) is the pion-nucleon form factor, which is commonly taken to have the form

$$v(t) = t(1+t^2/a^2)^{-1} \quad . \qquad (2.2)$$

The point is that most modern analyses which carefully treat the nucleon Born term show that a is a large number,[9,10] close to 1 GeV/c. Thus, it is favorable for the pion to develop large high momentum components in its wave function, particularly at low energy where the damping is relatively small. One also expects the evaluation of higher order terms in the optical potential to become more difficult because of the large value of a.

The second measure of the strength of the interaction is the size of the cross section, which becomes very large right at resonance. Here, we have

$$\bar{\sigma}_{TOT} = \tfrac{1}{2}\,[\sigma(\pi^+ N) + \sigma(\pi^- P)] \sim 130 \text{ mb}. \qquad (2.3)$$

If we determine the radius of the area seen by the pion by equating $\bar{\sigma}_{TOT}$ to πR^2 we find R = 2.0 fm. Comparing this to the average interparticle spacing, which at the center of the nucleus is about 2 fm, we see that the nucleons present strongly overlapping targets to the pion. It is

quite plausible to expect frequent scattering events in which the pion interacts with several nucleons at once, or with a "cluster" of nucleons. This makes higher order terms in the optical potential large and difficult to calculate.

There have, in fact, been numerous calculations of certain higher order terms in the optical potential over the last few years, and one term in particular has been identified as being particularly troublesome. The term in question is the local field correction, which arises from a scattering event in which the pion strikes the same nucleon twice, illustrated in Fig. 1. The most careful calculations of these have been given in references 4 and 11, but the term was also found troublesome in earlier works as well.[12] The local field correction appears to be able to attain a size comparable to the lowest order optical potential at the center of the nucleus.

Aside from the question of how to interpret the large isobar spreading interaction, the existence of the spreading potential raises additional questions of some practical importance. These are

1. What is the correct density dependence of the spreading interaction and U_2?

2. What is the spin and isospin dependence of the spreading interaction?

3. What is the spin, isospin and density dependence of the transition operator for inelastic scattering?

4. Can the pion be used as a tool for quantitatively probing the structure of nuclei?

It is hard to imagine that one can satisfactorally answer these questions unless more powerful theoretical methods are developed which permit reliable calculations beyond lowest order in the density expansion of the optical potential. The main subject of these seminars is to seek an answer to the question: what theoretical methods can permit reliable calculations when perturbation theory breaks down?, or put somewhat more generally: how can the many-body problem for hadron-nucleu

scattering be solved, given a fundamental interaction Hamiltonian?

III. Self-Consistent Hadron-Nucleus Scattering

The goal of self-consistency is to create a non-perturbative theoretical approach to hadron-nucleus scattering which will overcome the problems that arise because of the large strength of the elementary interactions. The idea was suggested in Ref. 13 and was developed there from a multiple scattering point of view. In this talk I will use an approach based on diagrammatic perturbation theory and the Green function $G(\underset{\sim}{r},\underset{\sim}{r}')$, defined as the amplitude to find a pion of energy ω at point $\underset{\sim}{r}$ in the medium if it was inserted in the medium at point $\underset{\sim}{r}'$. In the absence of interactions the Green function is given by

$$G_0(\underset{\sim}{r},\underset{\sim}{r}') = \frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{r}'|}}{4\pi|\underset{\sim}{r}-\underset{\sim}{r}'|} \tag{3.1}$$

where $\underset{\sim}{k}$ is the pion momentum. When interactions between the pion and the medium are considered, the interacting Green function G is given as the solution of an integral equation

$$G(\underset{\sim}{r},\underset{\sim}{r}') = G_0(\underset{\sim}{r},\underset{\sim}{r}') + \int d\tau_1 \int d\tau_2\, G_0(\underset{\sim}{r},\underset{\sim}{r}_1)\, U(\underset{\sim}{r}_1,\underset{\sim}{r}_2)\, G(\underset{\sim}{r}_2,\underset{\sim}{r}') \tag{3.2}$$

where U is called the optical potential in multiple scattering theory and the pion "proper self energy" in field theory.

To explain the idea of self-consistency, let us assume that the rules for constructing U in terms of the free pion-nucleon scattering amplitude, f, and the free pion Green function are known

$$U = U[f, G_0, \rho, \ldots] \tag{3.3}$$

The dots indicate that other quantities must be also specified in a complete theory. Let us also assume that the rules for constructing the free pion-nucleon scattering amplitude in terms of the elementary interaction Hamiltonian h, and the free Green function is known,

$$f = f[h_1, G_0, \ldots] \tag{3.4}$$

where we have again allowed for the theory to depend on other quantities as well. Usually the theory of scattering of a projectile from a composite

system is presented in terms of the rules we have assumed: one first solves the theory to obtain f, one secondly evaluates the optical potential in terms of this f and the density and lastly one solves for the scattering wave function to compare with the experimental data. For pions and presumably other strongly interacting probes as well, such as K^- and anti-protons, some steps in this link will not work. In the case of pions, the perturbation expansion for U in terms of f and ρ does not converge, which leads in turn to additional difficulties.

The idea of self-consistency is to provide a feedback mechanism in the chain of calculations so that the theory is given a way to dynamically modify itself in response to a pathological situation (for example an exceedingly strong projectile-nucleon interaction). Whether it will respond in the correct way is not à priori assured, and in any given case one has to look carefully at the results to convince oneself that the self-consistent theory is behaving better than the usual theory.

To create the feedback, the idea is to define a "self-consistent" amplitude, $\tilde{f}$, by substituting in Eq. (3.4) the complete Green function for G_0, i.e., define

$$\tilde{f} \equiv f[h_1, G, \ldots] \tag{3.5}$$

Now, such a substitution would lead to complete nonsense if G were defined as $(\omega-H)^{-1}$, which is a many-body operator, and we were working within the Watson or KMT multiple scattering formulations.[3] However, this substitution is allowed here because the Green's function of Eq. (3.2) is essentially defined so substitutions such as the one in Eq. (3.5) is permissible, i.e., it accounts for all possible "dressings" of intermediate pion propagators in f by the self energy effects due to the medium.

The second step of the transition to a fully self-consistent theory is to evaluate all the contributions to U in terms of $\tilde{f}$ and G. Thus, one insists that

$$U[f, G_0, \ldots] = \tilde{U}[\tilde{f}, G, \rho, \ldots] \tag{3.6}$$

and in order to have this equality satisfied the rules for enumerating

and evaluating diagrams must be carefully thought out in order to avoid over-counting or under-counting the terms, i.e., in order that the self-consistent theory is identical to the usual theory term-by-term in perturbation theory. The equation for G is still that given by Eq. (3.2).

The procedure for implementing self-consistency has now been fully specified. It is sufficiently general to be applied to any hadron-nucleus scattering situation which has a Hamiltonian formulation. Next, I want to give as an example the realization of these equations in a simple but nontrivial model of pion-nucleus scattering and to evaluate the conjecture that self-consistency leads to a more sensible theoretical description in this model. If this conjecture is valid, then one may study in detail the questions raised earlier, which have no well-defined answers in usual theories.

IV. Realization of Self-Consistent Theory in a Model

It may not be apparent to everyone how to proceed to implement self-consistency based on the discussion of Sect. III, so in this section I want to present a model in which all the details are worked out. Because the main intention of this section is pedagogical, a simple theory is more to the point than a complicated and possibly more realistic theory. Hence, purely for purposes of illustration, I choose to apply self-consistency to the fixed scatterer multiple scattering theory of Foldy and Walecka.[14] The main assumptions of this theory are that a separable potential acts between the projectile and the target nucleons, and that the target nucleons remain fixed as the projectile scatters. The positions of the nucleons are eventually averaged with a nuclear density function. The pion Green function may be evaluated based on the developments in the original paper by Foldy and Walecka,[15] and the various contributions to the Green function are illustrated in Fig. 2. To further simplify the problem it will be assumed that the pion interacts in an infinite nuclear medium of uniform density ρ. The nucleons will be assumed to experience short-ranged (anti-) correlations which are described by the radial distribution function R(r) given by

$$R(\underset{\sim}{r}) = \theta(|\underset{\sim}{r}|-A) \quad , A = 0.5 \text{ fm} \tag{4.1}$$

Only correlations between successively struck nucleons are retained.

For the sake of definiteness, let me state the rules for evaluating the diagrams of the pion Green function. The enumeration of the diagrams has been discussed already in the caption to Fig. 2. These are the rules of the usual theory, i.e., in the absence of application of the self-consistency requirement.

1. Nucleons do not propagate and are denoted by an "x." Each struck nucleon contributes one factor of ρ.

2. Pions are represented by directed lines which indicate the flow of momentum.

   a. Each internal pion line (of momentum $\underset{\sim}{t}$) is assigned the value

   $$\frac{e^{i\Delta\underset{\sim}{x}\cdot\underset{\sim}{t}}}{k^2-t^2+i\eta}\ \theta(|\Delta\underset{\sim}{x}| - A) \tag{4.2}$$

   where $\Delta\underset{\sim}{x}$ is the distance between the nucleons, counted in the direction of the pion momentum and k is the "incident" pion momentum ($\omega^2=k^2+\mu^2$).

   b. Initial and final pion propagators contribute, respectively, $e^{i\underset{\sim}{K}'\cdot\underset{\sim}{x}_i}$ and $e^{-i\underset{\sim}{K}\cdot\underset{\sim}{x}_f}$.

3. Each scattering contributes a factor

$$-4\pi\rho f(\underset{\sim}{t},\underset{\sim}{t}') \tag{4.3}$$

where f has a separable form [see Eq. (2.1)] in each partial wave.

4. Integrate over all internal pion momenta and nucleon positions.

The input to the theory is an off-shell pion-nucleon scattering amplitude and a correlation length A; the diagram rules provide the link between this input and the result we want, which is an expression for the pion self-energy. Note incidently, that although it makes no sense to actually scatter pions <u>from</u> an infinite medium, the Green function is a well-defined quantity, since it describes the propagation of the pion from a source which can be properly imagined to be embedded in the medium.

Once one has a theory for the pion self-energy, including its dependence separately on energy, density and momentum, it is possible to make estimates for scattering from a finite nucleus by using this quantity in the Klein-Gordon equation in the local density approximation. The exact, but technically much more difficult, procedure is to formulate the problem with the correct density-dependence of a finite system from the outset.

I have just described the theory to which I will now proceed to apply the ideas of self-consistency. The application has been worked out in detail in Refs. 15 and 16.

Consider first the self-consistent amplitude $\tilde{f}$. We first need the expression for f in terms of V and G_0 [see Eq. (3.4)], which is just the Lippman-Schwinger equation in terms of the potential V

$$f(\underset{\sim}{t},\underset{\sim}{t}') = V(\underset{\sim}{t},\underset{\sim}{t}') + 4\pi\int \frac{d^3t''}{(2\pi)^3} V(\underset{\sim}{t},\underset{\sim}{t}'')\, G_0(t'')\, f(\underset{\sim}{t}'',\underset{\sim}{t}') \tag{4.4}$$

where $G_0(t) = (k^2 - t^2 + i\eta)^{-1}$. In accordance with Eq. (3.5) we write

$$\tilde{f}(\underset{\sim}{t},\underset{\sim}{t}') = V(\underset{\sim}{t},\underset{\sim}{t}') + 4\pi\int \frac{d^3t''}{(2\pi)^3}\int \frac{d^3t'''}{(2\pi)^3} V(\underset{\sim}{t},\underset{\sim}{t}'')\, G(\underset{\sim}{t}'',\underset{\sim}{t}''')\, \tilde{f}(\underset{\sim}{t}''',\underset{\sim}{t}') \tag{4.5}$$

Eliminating V in favor of f in Eq. (4.5) yields

$$\tilde{f}(\underset{\sim}{t},\underset{\sim}{t}') = f(\underset{\sim}{t},\underset{\sim}{t}') + 4\pi\int \frac{d^3t''}{(2\pi)^3}\int \frac{d^3t'''}{(2\pi)^3} f(\underset{\sim}{t},\underset{\sim}{t}'')\tilde{g}(\underset{\sim}{t}'',\underset{\sim}{t}''')\tilde{f}(\underset{\sim}{t}''',\underset{\sim}{t}') \tag{4.6}$$

where

$$\tilde{g}(\underset{\sim}{t},\underset{\sim}{t}') \equiv G(\underset{\sim}{t},\underset{\sim}{t}') - (2\pi)^3\, \delta(\underset{\sim}{t}-\underset{\sim}{t}')\, G_0(t) \tag{4.7}$$

The operation leading from Eq. (4.4) to Eq. (4.6) is standard in scattering theory and a similar operation is described in detail in Ref. 3.

Next we need an expression for the Green function. In discussing interactions in a correlated medium it is convenient to define an auxiliary Green function, which propagates the pion between scattering centers. This quantity is defined diagrammatically in Fig. 3 and is represented diagrammatically by two parallel lines. This diagrammatic series is summed by the following integral equation

$$\tilde{G}(\underset{\sim}{t},\underset{\sim}{t}';\underset{\sim}{x}_f-\underset{\sim}{x}_i) = (2\pi)^3\,\delta(\underset{\sim}{t}-\underset{\sim}{t}')\,\frac{e^{i\underset{\sim}{t}\cdot(\underset{\sim}{x}_f-\underset{\sim}{x}_i)}}{k^2-t^2+i\eta}\,R(\underset{\sim}{x}_f-\underset{\sim}{x}_i)$$

$$-4\pi\rho\int d^3x\,\frac{e^{i\underset{\sim}{t}\cdot(\underset{\sim}{x}_f-\underset{\sim}{x})}}{k^2-t^2+i\eta}\,R(\underset{\sim}{x}_f-\underset{\sim}{x})$$

$$\times\int\frac{d^3t''}{(2\pi)^3}\,\tilde{F}(\underset{\sim}{t},\underset{\sim}{t}'';\,\underset{\sim}{x}_f-\underset{\sim}{x})\,\tilde{G}(\underset{\sim}{t}'',\underset{\sim}{t}';\,\underset{\sim}{x}-\underset{\sim}{x}_i) \qquad (4.8)$$

Note that $\tilde{G}$ depends on the initial and final pion momenta and by the positions of the initial and final nucleons struck. We use the notation that the kernel of the Green function is $-4\pi\rho\tilde{F}$.

The relationship between $\tilde{g}(\underset{\sim}{t},\underset{\sim}{t}')$ in Eq. (4.7) and $\tilde{G}$ of Eq. (4.8) is straightforward,

$$\tilde{g}(\underset{\sim}{t},\underset{\sim}{t}') = \tilde{G}(t,t';\,\underset{\sim}{x}_f-\underset{\sim}{x}_i)\Big|_{\underset{\sim}{x}_f=\underset{\sim}{x}_i} \qquad (4.9)$$

If we had not been using the correlations in the medium which vanish when $\underset{\sim}{x}_f = \underset{\sim}{x}_i$ then the subtraction in Eq. (4.7) would need to be made after setting $\underset{\sim}{x}_f = \underset{\sim}{x}_i$. The fact that R vanishes in our case means that the correct relationship is that of Eq. (4.9).

Finally, consider the series expansion for the kernel $\tilde{F}$ of the auxiliary Green function. The idea is to rearrange the diagrams which appear in the kernel U of the usual theory* so that these diagrams can be expressed explicitly in terms of $\tilde{f}$ and $\tilde{G}$ (and hence only implicitly on f and G_0). As stated in Sect. III, we insist that the diagrams of U expressed in terms of f and G_0 are in one-to-one correspondence with the diagrams of $\tilde{U}$, when $\tilde{f}$ and $\tilde{G}$ are expanded out in terms of f and G_0. Hence, to avoid making counting mistakes, some of the topologies which originally contributed to U will not contribute to $\tilde{U}$. The rules for constructing $\tilde{U}$ are

1. Begin by considering the expansion of U in terms of f and G_0. Eliminate all diagrams in which any intermediate pion propagator has a

*The kernel of the usual theory is, as stated earlier, the pion proper self-energy. More specifically, it is the sum of all diagrams beginning and ending with a single pion such that no diagram would break into two pieces when an internal pion line is cut.

proper self-energy insertion.

2. Replace all internal pion propagators by $\tilde{G}$.

3. Replace all free pion-nucleon scattering amplitudes f by $\tilde{f}$.

Figure 4 illustrates valid and invalid contributions to $\tilde{U}$. The terms (b), (e) and (f) do not exist in $\tilde{U}$ according to rules 1 to 3 above. Term (4b) is not valid because $\tilde{f}$ already contains all intermediate interacting pion propagators [see Eq. (4.5) and, for more detail, the argument presented in Sect. V.]. Term (4f) is not allowed because an intermediate pion propagator has a self-energy insertion. Diagram (4f) is already included in diagram (4c), since $\tilde{G}$ contains, by definition, all proper self-energy insertions. Diagram (4e) is not allowed for both reasons just discussed.

The self-consistent Foldy-Walecka theory is summarized in Fig. 5. Figure 5a shows the integral equation for $\tilde{G}$; Fig. 5b shows the integral for $\tilde{f}$; and, Fig. 5c gives the expression for the kernel $-4\pi\tilde{F}$. Note that the diagrams of $\tilde{F}$ depend on the density ρ implicitly through $\tilde{f}$ and $\tilde{G}$ and also explicitly, i.e., each vertex contains one explicit factor of ρ.

V. Solution of the Self-Consistent Theory, First Order

To solve the self-consistent theory summarized in Fig. 5, it is useful to proceed systematically, as follows. Group together the terms of Fig. 5c having the same number of explicit powers of density. We shall define order as the number of explicit powers of ρ appearing in the diagrams. Thus the first term on the right hand side of Fig. 5c is the first order term. The next three diagrams are the leading terms in the second order theory, i.e., the terms in the second order theory having the fewest number of factors of $\tilde{f}$ and $\tilde{G}$. The leading third order term is the last diagram shown in Fig. 5c. In this section we shall present results for the calculation of the first order theory in some detail and then present the results for the second order theory in Sect. VI.

The first order theory consists of the following equations

$$\tilde{F}(t,t';\Lambda x) \sim \tilde{f}(t,t')\,\delta^{(3)}(\Lambda x) \qquad (5.1)$$

where the $\delta^{(3)}(\Delta x)$ means that the initial and final pions both attach to the same nucleon,

$$\tilde{G}(\underset{\sim}{t},\underset{\sim}{t}';\ \underset{\sim}{x}_f-\underset{\sim}{x}_i) = (2\pi)^3\,\delta(\underset{\sim}{t}-\underset{\sim}{t}')\,\frac{e^{i\underset{\sim}{t}\cdot(\underset{\sim}{x}_f-\underset{\sim}{x}_i)}}{k^2-t^2+i\eta}\;R(\underset{\sim}{x}_f-\underset{\sim}{x}_i)$$

$$-\,4\pi\rho\int d^3x\,\frac{e^{i\underset{\sim}{t}\cdot(\underset{\sim}{x}_f-\underset{\sim}{x}_i)}}{k^2-t^2+i\eta}\,R(\underset{\sim}{x}_f-\underset{\sim}{x})\int\frac{d^3t''}{(2\pi)^3}\,\tilde{f}(\underset{\sim}{t},\underset{\sim}{t}'')\,\tilde{G}(\underset{\sim}{t}'',\underset{\sim}{t}';\ \underset{\sim}{x}-\underset{\sim}{x}_i) \tag{5.2}$$

where R(x) is given in Eq. (4.1), and

$$\tilde{f}(\underset{\sim}{t},\underset{\sim}{t}') = f(\underset{\sim}{t},\underset{\sim}{t}') - 4\pi\int\frac{d^3t''}{(2\pi)^3}\int\frac{d^3t'''}{(2\pi)^3}\,f(\underset{\sim}{t},\underset{\sim}{t}'')\,\tilde{g}(\underset{\sim}{t}'',\underset{\sim}{t}''')\,\tilde{f}(\underset{\sim}{t}''',\underset{\sim}{t}') \tag{5.3}$$

where

$$\tilde{g}(\underset{\sim}{t}'',\underset{\sim}{t}) \equiv \tilde{G}(\underset{\sim}{t}'',\underset{\sim}{t};\ \underset{\sim}{x}_f-\underset{\sim}{x}_i)\Big|_{\underset{\sim}{x}_f=\underset{\sim}{x}_i} \tag{5.4}$$

Equations (5.2)and (5.3) are two coupled integral equations which must be solved to obtain $\tilde{f}$ and $\tilde{G}$. The coupling between the integral equations provides the feedback upon which lies the hope of finding a more sensible solution than that provided by the usual theory.

Before showing the numerical solution of these equations I first want to address the question: how much of the complete multiple scattering series is contained in $\tilde{f}$? To answer this question, first iterate Eq. (5.3) to obtain the series expressing $\tilde{f}$ in terms of f and $\tilde{n}$ shown in Fig. 6. Next, find the expansion for $\tilde{g}$ in terms of $\tilde{f}$ and G_0 shown in Fig. 7 by iterating Eq. (5.2) and bringing the ends of the propagators together as required by Eq. (5.4). Finally, by repeatedly inserting Fig. 6 and Fig. 7 into each other, the sequence shown in Fig. 8 is obtained. One easily sees that

(1) all diagrams of $\tilde{f}$ are valid contributions to the pion proper self-energy in the Foldy-Walecka theory, properly counted. This illustrates explicitly the point discussed in a more general context below Eq. (3.5).

(2) The troublesome local field correction is contained in $\tilde{f}$. This result is desirable, because one would like the lowest order approximation in a theory to contain the most significant terms.

(3) Some valid contributions to the optical potential are not contained in $\tilde{f}$. This observation is essentially already made in recognizing there are second order (and higher) terms present in Fig. 5c. Note, however, that the order of summation is different in this theory and other theories[12,14] since the local field correction is not a second order term in our scheme.

If we assume that the free pion-nucleon scattering amplitude consists of a single partial wave ℓ,

$$f(\underset{\sim}{t},\underset{\sim}{t}') = [v(t)/v(k)]\, f(0)\, [v(t')/v(k)]\, P_\ell(\hat{\underset{\sim}{t}}\cdot\hat{\underset{\sim}{t}}') \tag{5.5}$$

where P_ℓ is a Legendre polynomial and $f(0)$ is the forward, on-shell pion-nucleon scattering amplitude. Eqs. (5.2) and (5.3) imply that $\tilde{f}$ has the form

$$\tilde{f}(\underset{\sim}{t},\underset{\sim}{t}') = [v(t)/v(k)]\, \tilde{f}(0)\, [v(t')/v(k)]\, P_\ell(\hat{\underset{\sim}{t}}\cdot\hat{\underset{\sim}{t}}') \tag{5.6}$$

where $\tilde{f}(0)$ is a function of energy and density. In other words, self-consistency does not change the functional form of the dependence of the amplitude on t or t', or its angular dependence; it changes only the energy-dependent term in the amplitude.

The quantity $\tilde{f}(0)$ can be shown[16] to satisfy the following integral equation, equivalent to Eqs. (5.2) and (5.3)

$$\tilde{f}(0) - f(0) = \frac{4\rho}{2\ell+1}\, \tilde{f}^2(0) f(0) \sum_m \int_0^\infty \frac{K^2 dK\, P^2(kK;\ell\ell;m)}{(K^2-k^2-i\eta)\,(K^2-k^2-4\pi\rho\tilde{f}(0)P(kK;\ell\ell;m)} \tag{5.7}$$

where the function $P(kK;\ell\ell;m)$ depends only on the "known" quantities $R(r)$ and $v(t)$,

$$P(kK;\ell\ell;m) = (-)^m \sum_l \begin{pmatrix} \ell & \ell & l \\ m & -m & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell & l \\ 0 & 0 & 0 \end{pmatrix} P_l^{\ell\ell}(k,K) \tag{5.8}$$

where

$$P_L(k,K) = ik(K^2-k^2)\int_0^\infty r^2 dr\, j_L(Kr)\, H_L^{\ell\ell}(k;r)\, R(r) \tag{5.9}$$

and

$$H_L^{\ell\ell}(k;r) = \frac{2}{\pi ik}\int_0^\infty t^2 dt\, j_L(tr)\,\frac{v^2(t)}{v^2(k)} \tag{5.10}$$

To obtain the numerical results shown next we have used the form factor in Eq. (2.2) with a variety of choices of α. Figure 9 shows $\mathrm{Im}\tilde{f}(0)$ as a function of incident pion momentum and for $\rho = \rho_0 \equiv 0.16\ \mathrm{fm}^{-3}$, which is approximately the central density of nuclei. The solid curve is $f(0)$, shown for comparison. The value $\alpha = 3.75\ \mathrm{fm}^{-1}$ is the preferred value, as it corresponds to Ref. 9a. Figure 10 shows $\mathrm{Re}\tilde{f}(0)$. One notices from the figures that

$$|\tilde{f}(0)|/|f(0)| < 1 \tag{5.11}$$

whenever $f(0)$ is large, which implies that $\tilde{f}$ is a more efficient expansion parameter than f.

It is evident from Figs. 9 and 10 that $\tilde{f}$ approaches a definite value as $\alpha \to \infty$. This limit has been called the "Beg limit." All theories have this limit[14] provided they satisfy the condition that

$$v(r) = 0 \quad \text{for } r > A/2 \tag{5.12}$$

where $v(r)$ is the Fourier transform of the pion-nucleon form factor $v(k)$ and A is the minimum distance of separation between nucleons enforced by short range correlations. It turns out that the theory of Eqs. (5.2) and (5.3) has an analytical solution in this limit.[15] Some of the properties of this solution are

(1) The dependence of $\tilde{f}(0)$ on ρ has a square root branch point at $\rho = \rho - 0.05\ \rho_0$. This implies that the self-consistent theory is equal to the usual theory for $\rho < \rho_0$ but not for $\rho > \rho_0$, since the usual theory will not converge at these densities.

(2) The pion mean free path λ may never be less than approximately 3/4 of the interparticle spacing. At nuclear matter densities this

gives $\lambda \approx 1.5$ fm, in contrast to estimates based on lowest order theories[17] which give $\lambda \approx 0.5$ fm at resonance. As emphasized by T. Ericson in his lectures,[18] no reasonable theory can be expected to give rise to variations in the pion wave function on a scale substantially shorter than that set by the interparticle spacing.

(3) No Kisslinger anomaly can occur at any density. This is a welcomed result, since in essentially all other theories of pion-nucleus scattering the pionic wave-function develops spurious high momentum components at low energy (this is the symptom of the Kisslinger anomaly) due to the strong off-shell behavior of the (3,3) amplitude. Specific higher order terms tend to push around the energy and density at which the anomaly occurs, but self-consistency eliminates it once and for all.

Consider next the density-dependence of the self-consistent amplitude. In Fig. 11 the density-dependence of $\tilde{f}(0)$ is shown for the case $\alpha = 3.75$ fm^{-1} for incident pion energies between 150 and 180 MeV. The results in the resonance region have been parameterized by the following expression, valid for incident pion kinetic energy from 150 to 210 MeV,

$$\tilde{f}(\rho,k) = f(k) + \frac{\rho}{\rho_c} \frac{1}{1+\rho/\rho_c} [g_1(k) + \frac{\rho}{\rho_c} g_2(k)] \tag{5.13}$$

where $g_1(k)$ and $g_2(k)$ are given by

$$g_i(k) = \sum_j h_{ij} (k-1.4)^j \ , \ 1.3 \leq k \leq 1.63 \text{ fm}^{-1}. \tag{5.14}$$

With k expressed in fm^{-1}, the numbers h_{ij} are given in Table I. The best fit to the density dependence was obtained with

$$\rho_c = \rho_0/4. \tag{5.15}$$

It is apparent from Fig. 11 and Eq. (5.13) that the rate of fall-off of $\tilde{f}$ is different for $\rho < \rho_c$ and $\rho > \rho_c$. At low density we have

$$\tilde{f}(\rho,k) \approx f(k) + \frac{\rho}{\rho_c} g_1(k) \qquad \rho < \rho_c \tag{5.16}$$

and at high density

$$\tilde{f}(\rho,k) \approx f(k) + g_1(k) + \frac{\rho}{\rho_c} g_2(k) \qquad \rho >> \rho_c \tag{5.17}$$

For pion elastic scattering near resonance, the angular distribution is determined mostly by the property of the optical potential near $\rho/\rho_0 = 0.1$,[19] and therefore Eq. (5.16) determines the ρ^2 correction to U (recall $U = -4\pi\rho\tilde{f}$) which is effective in elastic scattering. The correction is small due to the fact that ρ is small, but it is nevertheless larger than one might ordinarily expect for a ρ^2 term. For example, at $k = 1.4$ fm^{-1} we easily find (see Table I)

$$\left| \frac{\rho_0}{\rho_c} \frac{g_1(1.4)}{\tilde{f}(1.4)} \right| = 1.6 \qquad (5.18)$$

This is unacceptably large in a theory which constructs U from terms linear and quadratic in the density only, but in the theory being discussed a catastrophy at the center of the nucleus is avoided by the higher order terms in $\tilde{f}$.

VI. Solution of the Self-Consistent Theory, Second Order

Here I will skip most of the details of the calculation; they can be found in Ref. 16. The idea is to sum the second order terms shown in Fig. 5c. Fully self-consistent $\tilde{f}$ and $\tilde{G}$ were used to evaluate these terms in accordance with the theory of Sects. III and IV. The numerical results for the second order terms are shown in Figs. 12 and 13. The size of the correction tends to be larger for larger α. Figure 14 shows the relative size of the second order piece to the sum of first plus second order. For a realistic choice of α, corresponding to the long dashes, the correction is less than 20% for $k/\mu > 1.4$ or pion kinetic energy greater than 100 MeV. Thus, for the purpose of studying scattering of pions in the resonance region the lowest order self-consistent theory provides a semi-quantitative solution to the model problem.

What happens when $\delta\tilde{f}$ is added to $\tilde{F}$ and Eq. (4.5) is solved to obtain a new $\tilde{f}$? It is a matter of experience that the results are stable against this iteration. The source of this stability can be seen as follows. Suppose that an additive correction λ is made to $\tilde{F}$. For the sake of illustration, let this correction have the same partial wave

component ℓ as f and $\tilde{f}$. According to the theory summarized in Fig. 5, $\tilde{f}$ must now be evaluated with the Green function $\tilde{G}$ which has the new $\tilde{F}$ as its kernel. In this way $\tilde{f}$ depends on λ, and we want to see how $\tilde{f}$ depends on λ. It is easily seen that $\tilde{f}(U,\lambda)$ now satisfies the equation [compare to Eq. (5.7)]

$$\tilde{f}(0;\lambda)=f(0) + \frac{4\rho}{2\ell+1}\,\tilde{f}(0;\lambda)[\tilde{f}(0;\lambda)+\lambda]\; f(0)\sum_m \int K^2 dK\; \frac{P^2(kK;\ell\ell;m)}{K^2-k^2-i\eta} \times \frac{1}{K^2-k^2-4\pi\rho[\tilde{f}(0;\lambda)+\lambda]\; P(kK;\ell\ell;m)} \tag{6.1}$$

Upon adding λ the quantity $\tilde{F}(0) = \tilde{f}(0)$ becomes

$$\tilde{F}(0) \to \tilde{F}(0;\lambda) = \tilde{f}(0;\lambda) + \lambda \tag{6.2}$$

For small changes λ, $\tilde{f}(0;\lambda)$ may be evaluated by making a Taylor series expansion. The derivative $d\tilde{f}/d\lambda$ may be evaluated using Eq. (6.1), and we find

$$\frac{d\tilde{f}}{d\lambda} = \frac{\rho}{f}\frac{d\tilde{f}}{d\rho} \tag{6.3}$$

and hence

$$\tilde{F}(0;\lambda) = \tilde{f}(0) + \lambda\left(1 + \frac{\rho}{f}\,\frac{d\tilde{f}}{d\rho}\right) \tag{6.4}$$

evaluating the correction near resonance [using Eq. (5.13) and Table I] and for $\rho = \rho_0$, we find that

$$\left| 1 + \frac{\rho}{f}\,\frac{d\tilde{f}}{d\rho} \right| \approx 0.75 - 0.85 \tag{6.5}$$

with the largest reductions occurring off-resonance. This reduction factor decreases very slowly as ρ is decreased. We therefore see that the response of the system is such as to oppose the addition of further corrections. Thus U and results which depend on U are less sensitive to corrections than might have been expected, when U is evaluated self-consistently. Corrections to the Foldy-Walecka theory which must be considered include binding and recoil of the nucleon and isobar, the spin-orbit force, the Pauli principle and the true absorption of the pion.

The main result of this section is the verification of the conjecture made in Sect. III that the expansion of the optical potential would converge rapidly when evaluated self-consistently. The reasons for expecting the same conclusion to hold at higher order are

(1) The use of $\tilde{f}$ rather than f as an expansion parameter to evaluate higher order terms in U. It was shown in Sect. V that $|\tilde{f}/f| < 1$ whenever f is large, and hence terms with large numbers of scatterings are strongly suppressed.

(2) The use of $\tilde{G}$ rather than G_0 to evaluate higher order terms in U. This is advantageous because $\tilde{G}$ falls off with distance exponentially in accordance with the mean-free path of the pion, with the result that terms with large numbers of interconnecting pion propagators are strongly suppressed.

VII. Comparison of Self-Consistent Theory to Experiment

Sections IV to VI concerned the pedagogical development of the idea of self-consistency in a model. The model has a number of shortcomings which were enumerated in the last section, and one might therefore expect that the numerical results would not compare favorably to experiment. However, we have seen that the sensitivity of the theory to addition of corrections is less strong than it would have been in the absence of self-consistency. In view of this I shall proceed to interpret some of the recent interesting experimental and theoretical results in terms of the framework developed here.

I first want to compare the results of the last two sections to the phenomenological spreading potential of Horokawa, Thies and Lenz,[7] discussed in Sect. I. To make the comparison, I define the spreading interaction W_0^I in the theory in the same way as done in Ref. 7, i.e., $\mathrm{Re}W_0^I$ determines the shift of the mass of the isobar and $\mathrm{Im}W_0^I$ determines the spreading of the width. This can be accomplished by setting

$$\tilde{f}(0) + \delta\tilde{f}(0) = f(0)\,\frac{\omega-\omega_R+i\Gamma(k)/2}{\omega-\omega_R-W_0^I+i\Gamma(k)/2} \tag{7.1}$$

where $\bar{f}(0)$ and $\delta\bar{f}(0)$ are the first and second order contributions to $\tilde{F}$ in accordance with Sect. V and VI with $\alpha = 3.75$ fm^{-1} and $\rho = \rho_0$. Equation (7.1) is solved for W_0^T and compared to the phenomenological result. This comparison is meaningful because neither W_0 nor W_0^T includes the Pauli effect, the spin-orbit potential of the isobar, and the binding and recoil of the isobar. True absorption of the pion is not explicitly included in the model but, as we argue below, a comparison to experiment is still meaningful. The results are shown in Fig. 15. It is seen that the sign and magnitude of the experimental result is reproduced by the theory. I regard this comparison to be remarkably good, considering that the theory contains no adjusted parameters (the value of α was taken from Ref. 9a). On the basis of this figure there appears to be more energy dependence in the experiment, but it should be observed that the empirical value of $\mathrm{Im}W_0$ does become positive at somewhat higher energies[1] for ^{16}O, which might be taken as evidence in favor of the shape of the energy dependence of the theory.

As we pointed out above, the true absorption of the pion is not explicitly taken into account in our model calculation. Yet it is undoubtedly true that a large part of the W_0 term is related to the true absorption of the pion. How can we reconcile the excellent agreement in Fig. 15 with this fact? Part of the answer is to be found in the following result. We have added a constant imaginary term proportional to ρ to the kernel $\bar{F}$ to simulate the true absorption of the pion. We found that the self-consistent value of $\bar{F}$ (and hence W_0^T) was very insensitive to the magnitude of this term, upon varying it from the size found in empirical pionic atom studies to 10 times this value. The insensitivity is understood at least in part from the argument presented at the end of the last section.

The physical interpretation of this result is the following. The depletion of flux from the elastic channel is already so complete due to quasielastic scattering, that there is little left for other channels to remove. For elastic scattering of a sufficiently strongly absorbed probe,

these results suggest that it is much more important to evaluate the interaction self-consistently than to treat the individual reaction channels in full detail.

To learn whether the dominant reaction mechanism in the model theory is approximately correctly described one would like to know the extent to which multiple quasielastic scattering plays a role in the total pion reaction cross section. It is commonly asserted that it plays a relatively unimportant role[20] because the partial cross section for a pion to disappear completely in the final state accounts for about half of the reaction cross section. However, this interpretation is called into question in a recent experimental result,[21] which finds evidence for clusters containing as many as five nucleons to be involved in the absorption process in a heavy nucleus. This result suggests that the true absorption cross section is driven by multiple quasielastic scattering, i.e., that true absorption is the inevitable demise of the pion after it does what it most likes to do, undergo multiple quasielastic collisions. It is tempting to associate these clusters with the higher order terms of Fig. 8, but a proper calculation of the absorption of the pion in such an involved event would require adopting a more comprehensive theoretical framework than that provided by the model theory on which our numerical results are based. It is important to come to a clearer empirical characterization of the reaction mechanism in pion-nucleus scattering and one would therefore like to have more detailed data of the type pioneered by Ref. 21.

To understand in somewhat greater detail the effect of the higher order corrections in U on elastic scattering we have evaluated the elastic scattering angular distributions based on the construction of the optical potential from the lowest order term of the theory of Sect. V. To display our results in a meaningful fashion, we have evaluated the parameters b_1, Y and a which reproduce the cross section in the form[22]

$$\frac{d\sigma}{d\Omega} = \left|\frac{kR}{2q} J_1(qR)\ \mathrm{Re}\ \Gamma(1-aq)\right|^2 \tag{7.2}$$

where

$$R^2 = b^2 + \frac{b}{Yq}\left[\Gamma(1-iaq-a/2b) - \Gamma(1+iaq-a/2b)\right]/\mathrm{Re}\ \Gamma(1-iaq) \tag{7.3}$$

and

$$\bar{b} = b_1 + a[\ln \ln 2 + \tfrac{1}{2} \ln(1+Y^2) - i \arctan Y] \qquad (7.4)$$

The parameter b_1 determines mostly the location of the diffractive minima, Y the depth of the minima and a the rate of fall-off of the angular distribution. The energy dependence of these quantities are shown in Figs. 16 to 18. The dots represent experimental points determined as described in Ref. 23. The dashed curves are the values obtained from the standard lowest order optical model theories and the solid curves represent the result of the self-consistent theory. Notice that the self-consistent theory is in slightly better agreement with the data than the lowest order theory. This presumably reflects the agreement with W_0 seen in Fig. 15. The fact that there remains a discrepancy, particularly evident in Fig. 17, between the data and the experiment is presumably a reflection of the fact that the energy shift arising from the recoil of the isobar has been neglected in constructing U.[24] The higher order terms have measurable effects on the cross section in the resonance region, and a proper description of the scattering requires inclusion of both the higher order terms and the energy shift due to the isobar recoil.

To obtain the results in Figs. 16 to 18 the optical potential was constructed from the densities based on Ref. 25. The optical potential utilizes the density dependence of $\tilde{f}$ in Eq. (5.13) and Table I and assumes that the off-shell extrapolation of $\tilde{f}$ is linear in the initial and final pion momenta. A local representation of U was obtained and then the analytical theory of Ref. 22 was used to calculate b_1, a and Y from this potential.

VIII. Summary and Discussion

The idea of self-consistent evaluation of hadron-nucleus scattering amplitudes was developed in a general context in Sect. III and illustrated in a model in Sects. IV to VI. The main motivation of self-consistency was to search for an alternative method for evaluating scattering from nuclei theoretically when the underlying hadron-nucleon interaction is very strong. We were able to verify in a nontrivial model of pion-

nucleus scattering that the self-consistent evaluation of the theory provides a solution when usual approaches fail. Similar methods would presumably be of value in studies of K^--nucleus and anti-proton nucleus scattering.

In Sect. II a number of theoretical questions were posed. These questions have remained essentially unanswered because of the convergence difficulties encountered in calculating the pionic optical potential beyond the lowest order. The self-consistent theory does not have these difficulties and these questions can therefore be addressed in any given model. In Sect. VII I examined in some detail the interpretation of the spreading interaction of Ref. 2 using the results of Sects. IV to VI. The main result is that the theory reproduces the sign and magnitude of the spreading interaction with no adjusted parameters, and that this result appears to be very insensitive to details of the model as a consequence of the self-consistent treatment.

Recently several calculations of pion-nucleus scattering have been reported,[26] which represent ambitious extensions of self-consistency as applied in Sects. IV to VI, and in which full recoil, Pauli effects and true absorption are taken into account. These calculations are much more complicated than the one presented here. The insensitivity of the calculation to addition of higher order effects discussed in Sect. VI gives rise to the conjecture that the corrections such as those attempted in Ref. 26 will have a small effect. To check this conjecture, one would like to see a self-consistent calculation with all the corrections included compared to a self-consistent calculation with the corrections selectively turned off. If the theory proves to be insensitive to corrections when evaluated in this fashion, we would then be much closer to a practical, microscopic theory of pion-nucleus interactions.

The second question examined in some detail was the density dependence of the optical potential. The self-consistent theory gives rise to a new result, namely it predicts the existence of a critical density, ρ_c, which we find to occur at $\rho_c = \rho_0/4$. For densities less than ρ_c the

density-dependence is quantitatively different from that for densities greater than ρ_c. The most rapid density variation occurs for low densities, on the basis of which one may characterize the higher order effects as sticking out in the nuclear surface. It is therefore necessary to include the density-dependent corrections in the optical potential arising from the self-consistent treatment of scattering. The effect of these corrections on the angular distribution was examined in Sect. VII.

The strong density dependence of the higher order corrections in the nuclear surface has implications for phenomenological analyses of scattering experiments. In most analyses the higher order corrections were assumed to have a smoother density variation than the one we find in the model analysis. If the density variation were allowed to be stronger in the surface than in the nuclear interior, it would not be surprising to see quantitatively different answers emerging in Refs. 1, 2, 7, and 8.

Several additional questions were raised in Sect. II which were not answered here, but could easily be addressed on the basis of the methods developed. These questions relate to the spin and isospin dependence of the self-consistent theory. Answers to such questions are needed in order to extend the theory to treat inelastic and charge exchange scattering The equations needed to evaluate these effects in the Foldy-Walecka theory are straightforward generalizations of the results of Refs. 15 and 16. The successes of this model as discussed in Sect. VII make such a calculation very interesting at the present time. Particularly in the case of single and double charge exchange to analog states, the higher order terms to the optical potential have a striking effect,[27] and these reactions may afford the best opportunity for a systematic study.

Finally, the question was raised as to whether pion-nucleus scattering can be used to learn about details of nuclear structure. This question will continue to be controversial for years to come, but the results of the present theory have some discouraging and some encouraging implications. The discouraging result is that the higher order corrections stick out into the nuclear surface, where one has hoped to be able to see the

effects of nuclear structure without complications arising from higher order effects. The encouraging result is that the simple self-consistent approach discussed here seems to work well and may therefore be a useful theoretical approach to serious microscopic calculations of pion- (and other hadron-) nucleus scattering.

Table I. Values of h_{ij} in Eq. (5.14)

j	h_{1j}	h_{2j}
0	(0.064,-0.49)	(0.006,-0.041)
1	(3.50, 1.46)	(0.038,-0.265)
2	(-9.80,22.3)	(2.50,-2.49)
3	(-14.1,-78.1)	(-3.06,16.7)

Figure Captions

1. Diagrammatic representation of the local field correction. Figure 1a gives the representation in terms of the diagrams of Foldy and Walecka, used extensively in these lectures, and Fig. 1b gives the representation in terms of Feynman-Goldstone diagrams. In the local field correction a nucleon (denoted 1) is struck twice with an intervening collision with nucleon 2.
2. Illustrating contributions to the pion Green function in the theory of Foldy and Walecka. Two types of processes are allowed: the pion may multiply scatter without ever hitting the same nucleon more than once, or it may come back to a given nucleon after striking at least one other nucleon before doing so.
3. Diagrammatic definition of the auxiliary Green function, $\tilde{G}$. The blobs are the pion proper self-energy insertions. The open circles are to remind one that $\tilde{G}$ ends and begins on scattering centers.
4. Illustrating valid and invalid contributions to $\tilde{\Gamma}$. The double solid lines are $\tilde{G}$, defined in Fig. 3. The triangle A is the diagrammatic representation of $\tilde{f}$, defined in Eq. (4.6). Terms (b), (e), and (f) are not allowed in $\tilde{F}$ as they would lead to double counting.
5. Summary of the fully self-consistent theory in diagrammatic notation.
6. Diagrammatic expansion of the self-consistent amplitude $\tilde{f}$ (A) in terms of f (x) and $\tilde{g}$.
7. Diagrammatic expansion of the interacting Green function $\tilde{g}$ in terms of $\tilde{f}$ and G_0.
8. Diagrammatic content of $\tilde{\Gamma}$ in terms of f and G_0 in lowest order theory.
9. $\mathrm{Im}\tilde{f}(0)$ for various values of α at $\rho = 0.16$ fm^{-3}. The legend is ---- $\alpha = 2.5$ fm^{-1}; - - - - $\alpha = 3.75$ fm^{-1}; — — — $\alpha = 8.5$ fm^{-1}, ····· $\alpha = \infty$. The solid curve is f(0).
10. $\mathrm{Re}\tilde{f}(0)$ for various values of α at $\rho = 0.16$ fm^{-3}. The legend is the same as for Fig. 9.

Figure Captions (Cont'd.)

11. Density dependence of $\tilde{f}$ in the resonance region. The upper set of curves is $\mathrm{Im}\tilde{f}(0)$ for T_π = 150 to 180 MeV. The lower set corresponds to $\mathrm{Re}\tilde{f}(0)$. The calculation assumes $\alpha = 3.75\ \mathrm{fm}^{-1}$.

12. Imaginary part of the second order correction to $\tilde{F}$. The legend is the same as Fig. 9.

13. Real part of the second order correction to $\tilde{F}$. The legend is the same as Fig. 9.

14. Relative size of the second order correction to $\tilde{F}$. The legend is the same as Fig. 9.

15. Comparison of the theoretical spreading potential, W_0^T, to the phenomonological result of Ref. 2. The solid curve is value W_0^T obtained from Eq. (7.1) and the points with the error bars are taken from Ref. 2. The triangles come from an analysis of ^{4}He, the squares ^{16}O and the circles ^{12}C.

16. b_1 vs. pion kinetic energy T_π in the lowest order free (- - -) and self-consistent theories (-----) for ^{40}Ca. The dot is the empirical value taken from Ref. 23.

17. Y vs. pion kinetic energy T_π. The legend is the same as Fig. 16.

18. $\underline{a}$ vs. pion kinetic energy T_π. The legend is the same as Fig. 16.

References

1. M. Hirata, F. Lenz and K. Yazaki, Ann. Phys. 108, 116 (1977); M. Hirata, J.-H. Koch, F. Lenz and E. J. Moniz, Phys. Lett. 70B, 281 (1977); Ann of Phys. 120, 205 (1979).
2. Y. Horikawa, M. Thies and F. Lenz, MIT preprint CTP 844, MAR 1980.
3. H. Feshbach, Lectures presented at this summer school.
4. E. Oset and W. Weise, Phys. Lett. 77B, 159 (1978); Nucl. Phys. A318 477 (1979).
5. K. Klingenbeck, M. Dillig and M. G. Huber, Phys. Rev. Lett. 41, 387 (1978); H. M. Hoffman, Z. Physik A289, 273 (1979).
6. A. L. Fetter and J. D. Walecka: Quantum Theory of Many-Particle Systems (New York, 1971).
7. K Stricker, H. McManus and J. A. Carr, Phys. Rev. C19, 929 (1979).
8. L. C. Liu and C. M. Shakin, Phys. Rev. C19, 129 (1979); L. C Liu, Phys. Rev. C17, 1778 (1978).
9. (a) D. J. Ernst and M. B. Johnson, Phys. Rev. C17, 247 (1978); (b) D. J. Ernst and M. B. Johnson, Phys. Rev. C (to be published).
10. C. A. Dominguez and R. B. Clark, Phys. Rev. C21, 1944 (1980); C. A. Dominguez and B. J. Verwest, Phys. Lett. 89B, 333 (1980), M. Dillig and M. Brack, J. Phys. G, Nucl. Phys. 5, 223 (1979).
11. M. K. Banerjee and S. J. Wallace, Phys. Rev. C21, 1996 (1980).
12. B. D. Keister, Nucl. Phys. A271, 342 (1976); M. Thies, Phys. Lett. 63B, 39 (1976).
13. R. M. Frank, J. L. Gammel and K. M. Watson, Phys. Rev. 101, 891 (1956).
14. L. L. Foldy and J. D. Walecka, Ann. of Phys. 54, 447 (1969).
15. M. B. Johnson and H. A. Bethe, Nucl. Phys. A305, 418 (1978).
16. M. B. Johnson and B. D. Keister, Nucl. Phys. A305, 461 (1978).
17. J. Hüfner, Phys. Reports 21C (1975).
18. T.E.O. Ericson, lectures presented at this school.
19. M. B. Johnson and H. A. Bethe, Comm. Nucl. Part. Phys. B, 75 (1978).

References (Cont'd.)

20. F. Lenz in High Energy Physics and Nuclear Structure, edited by M. P. Locker (Basel, 1977), p. 175.

21. R. D. McKeown, S. J. Sanders, J. P. Schiffer, H. E. Jackson, M. Paul, J. R. Spect, E. J. Stephenson, R. P. Redwine and R. E. Segel, preprint.

22. This representation of the elastic amplitude has been studied in detail by J.-F. Germond and M. B. Johnson, Phys. Rev. C (1980).

23. (a) J.-F. Germond and M. B. Johnson, contribution to the 8th Intern. Conf. on High Energy Physics and Nuclear Structure, Vancouver, B.C., 1979.

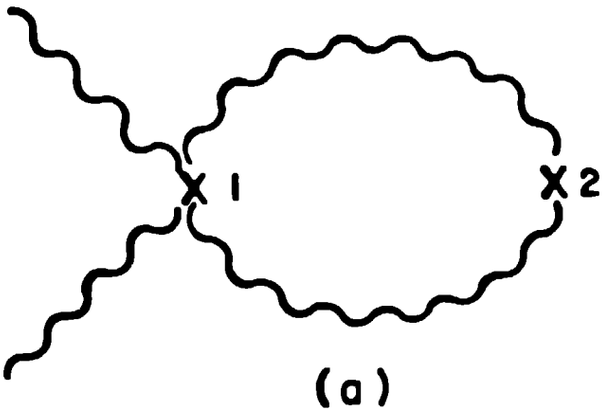
    (b) M. B. Johnson in Workshop on Nuclear Structure with Intermediate Energy Probes, Los Alamos Report LA-8303-C, J. 1980, p. 130.

24. R. Amado, F. Lenz and K. Yazaki, Phys. Rev. C18, 918 (1978).

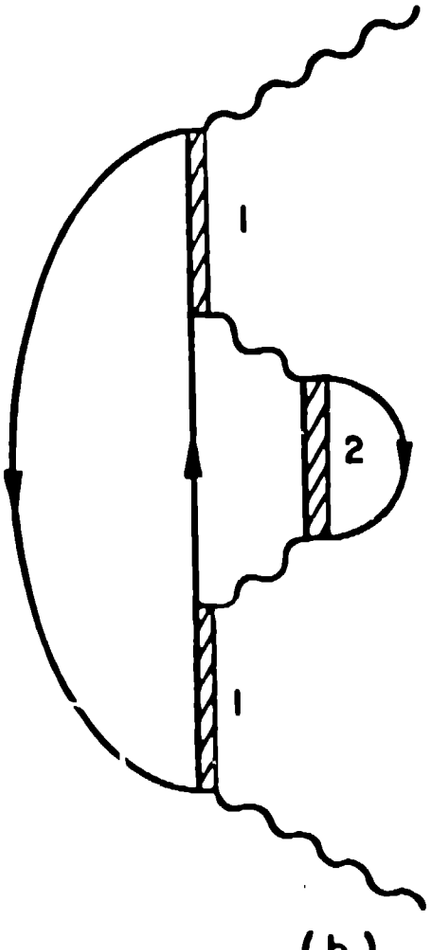
25. J. W. Negele and D. Vautherin, Phys. Rev. C5, 1472 (1972); Phys. Rev. C11, 1031 (1975).

26. J. W. Van Orden, M. K. Banerjee, D. M. Schneider and S. J. Wallace, Bull. Am. Phys. Soc. 25 Abstract DE 5 (1980); C. Schmit, preprint; R. Cenni and G. Dillon, lectures presented at this summer school.

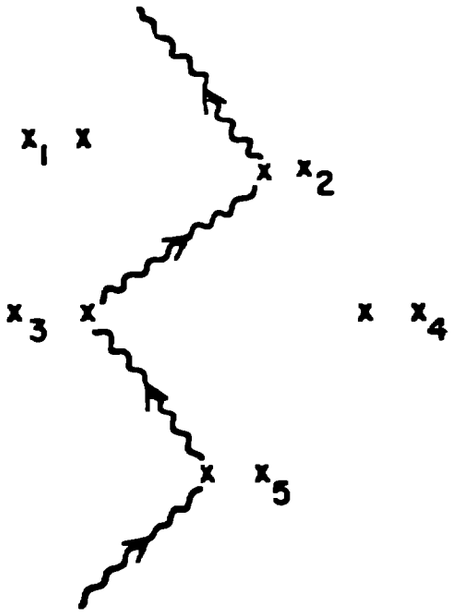
27. G. Miller and J. Spencer, Ann. of Phys. 100, 562 (1976); M. B. Johnson and E. Siciliano, to be published.



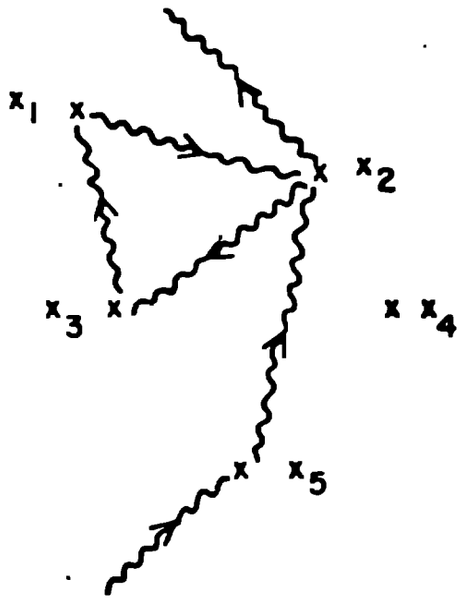
(a)



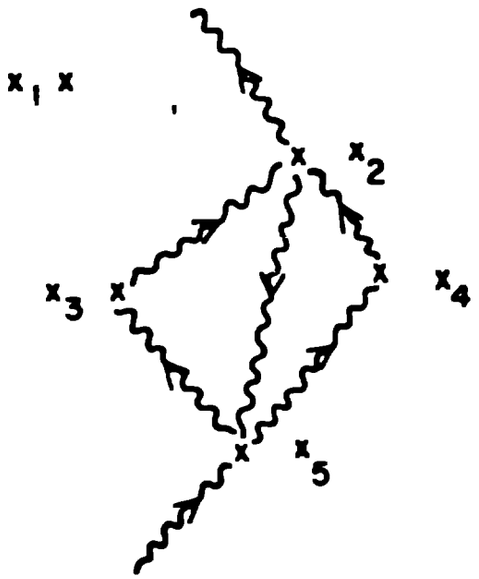
(b)



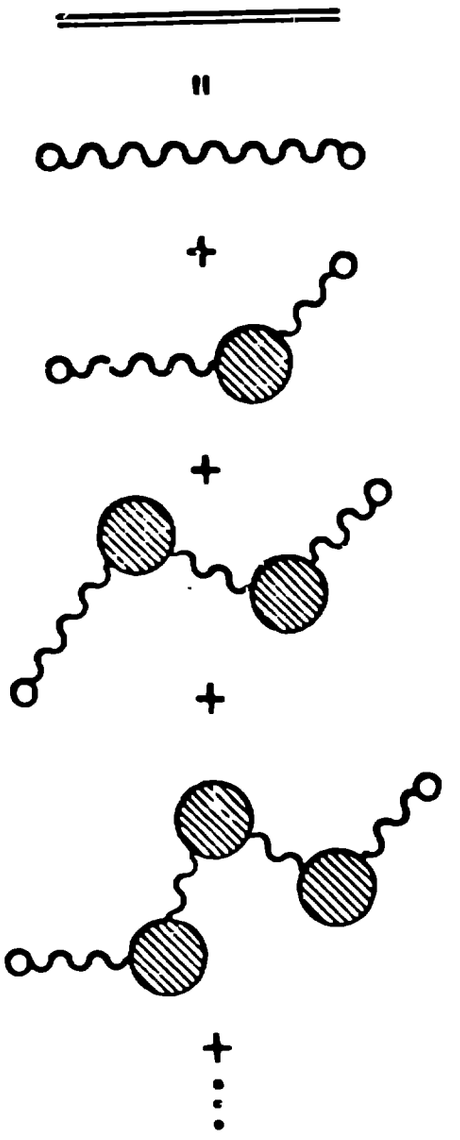
(A)

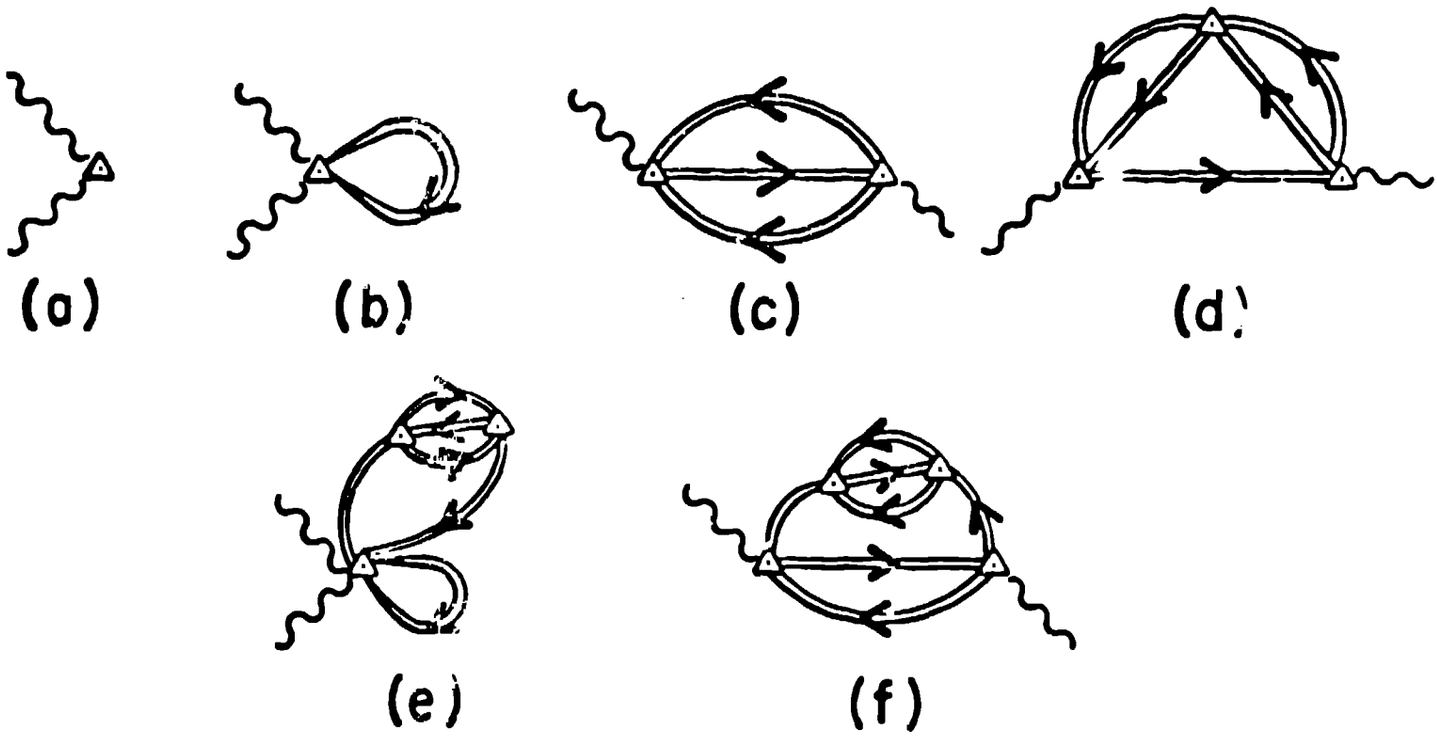


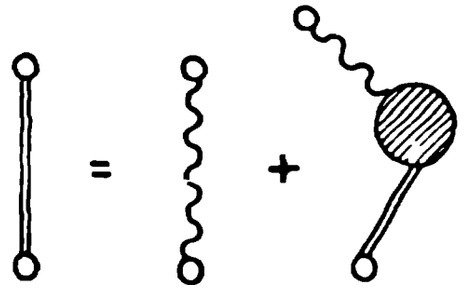
(B)



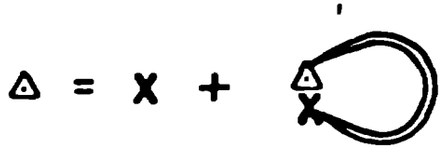
(C)



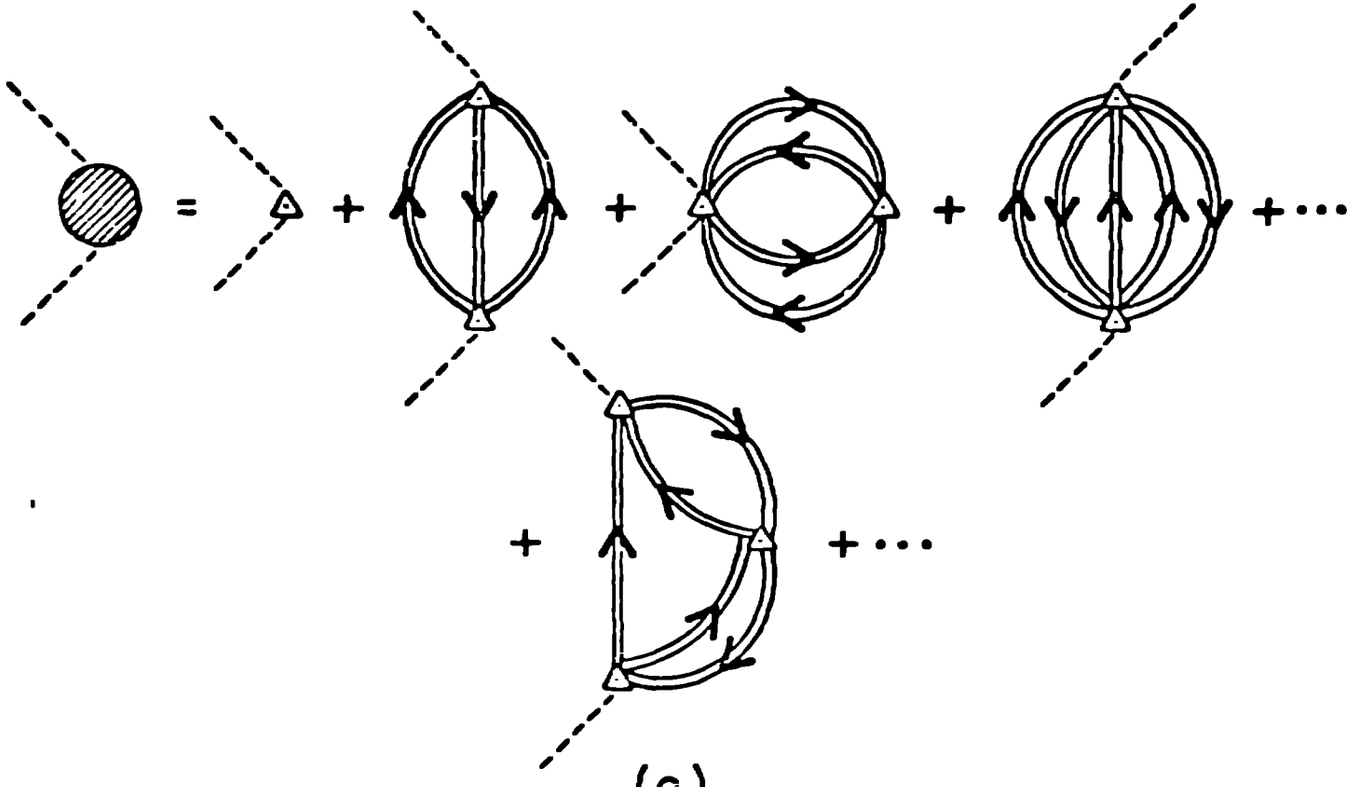

(a)
(b)
(c)
(d)
(e)
(f)



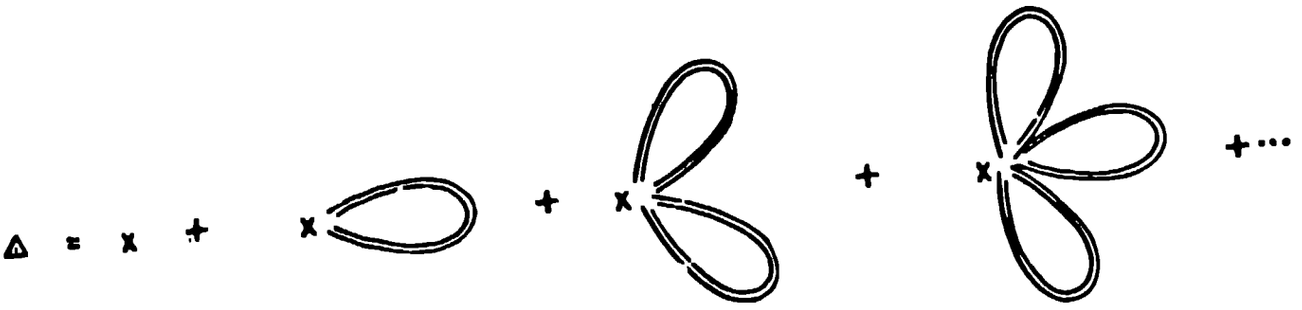
(a)

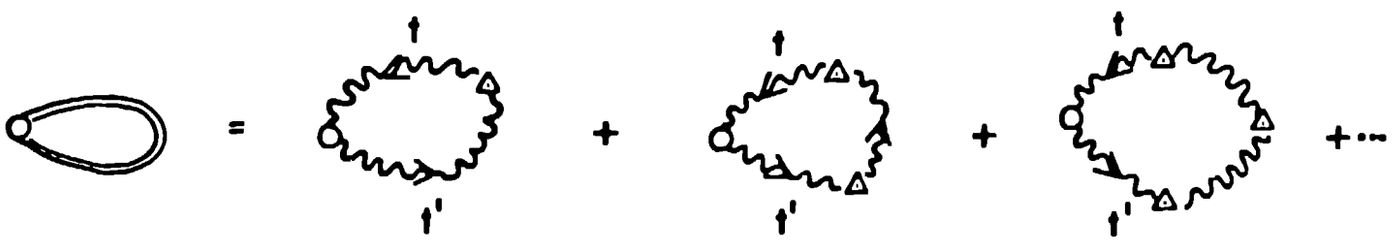


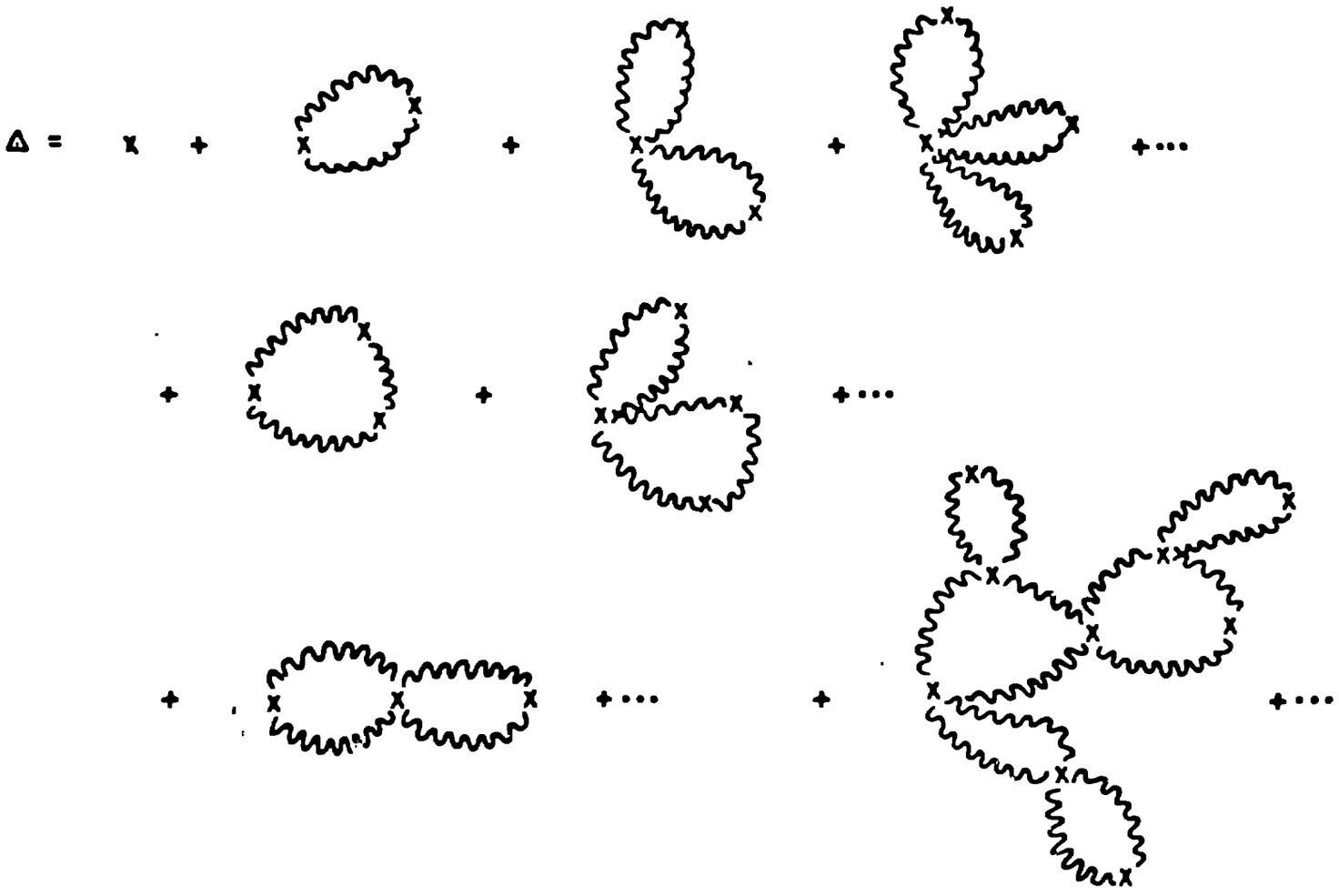
(b)

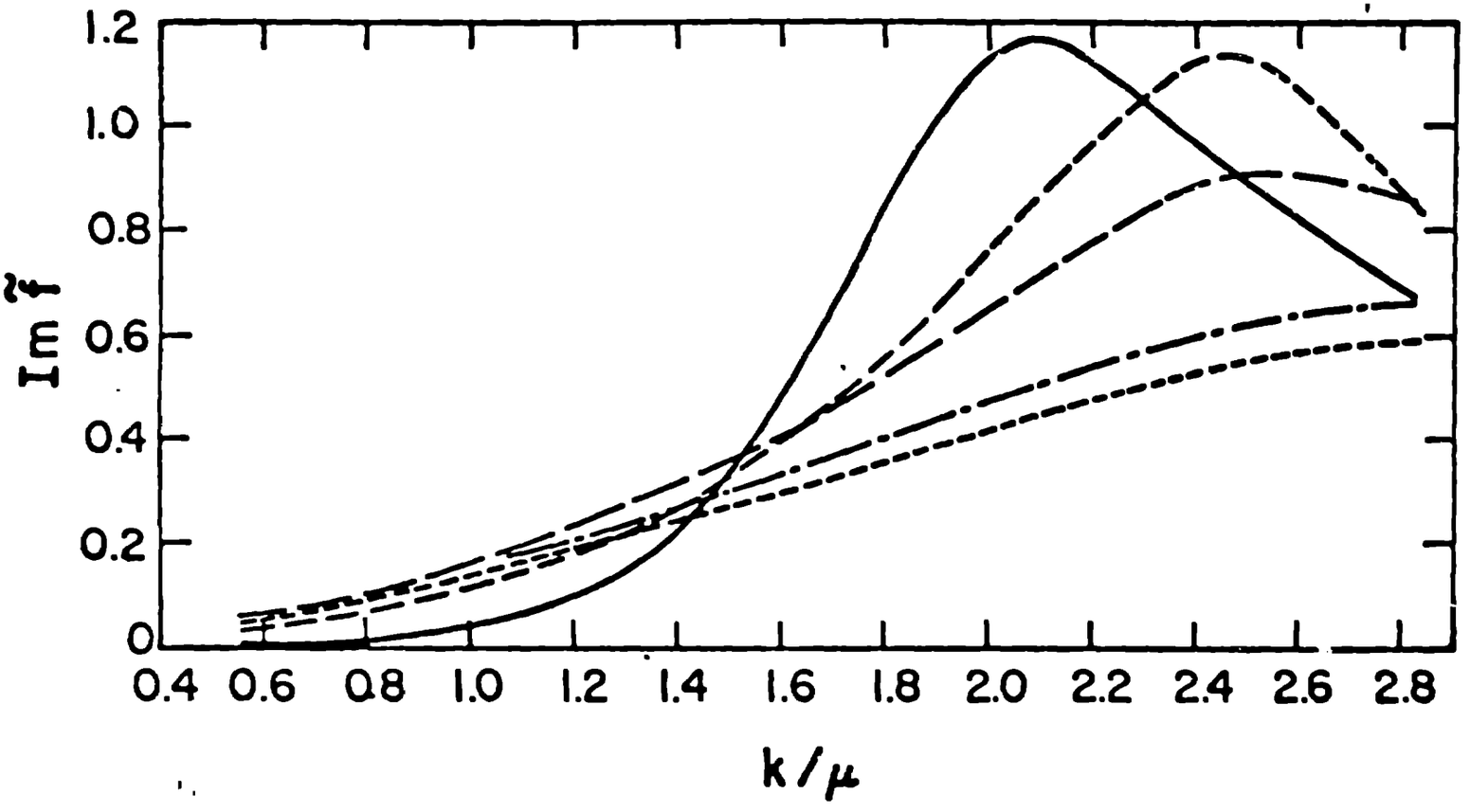


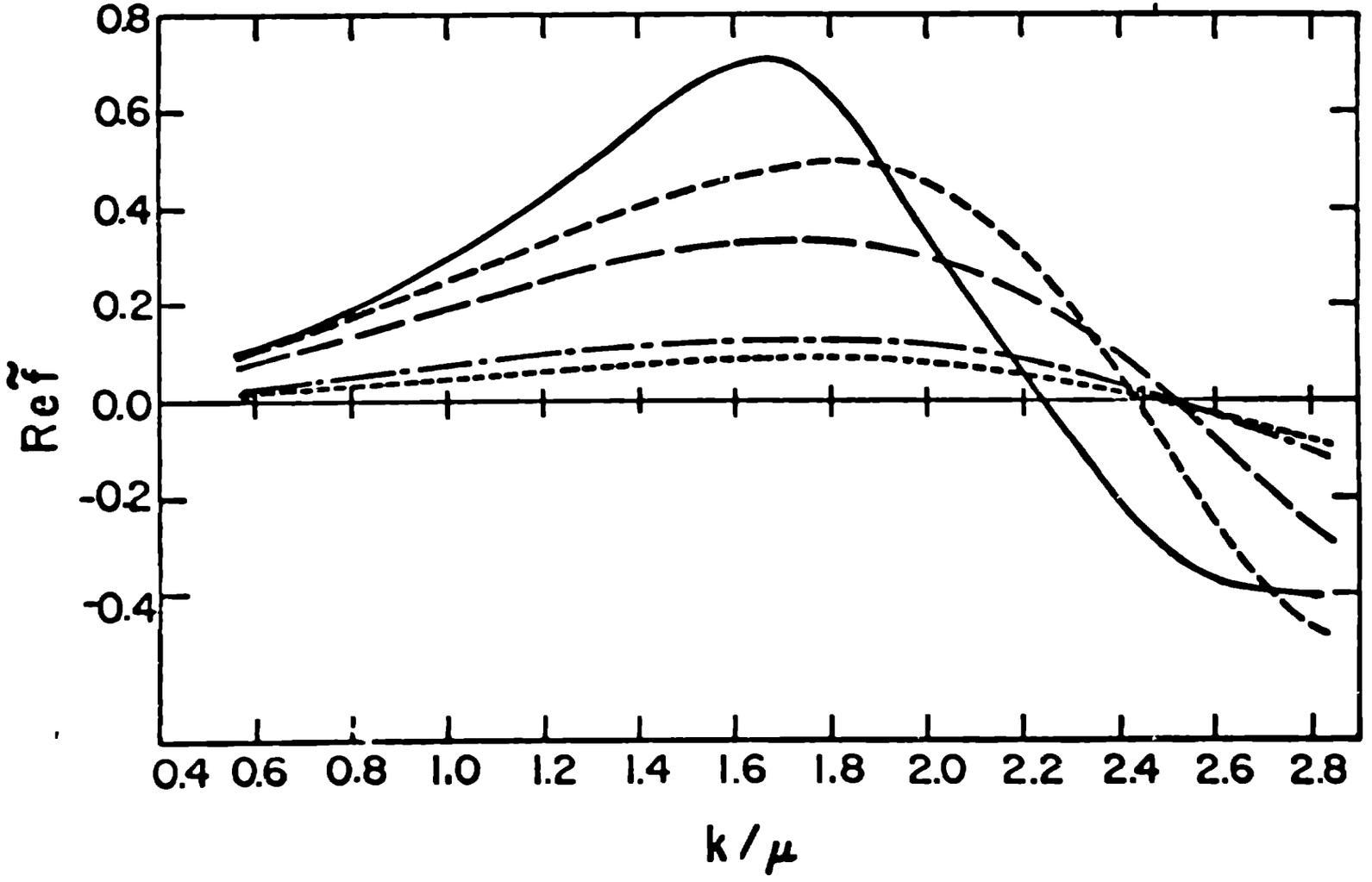
(c)

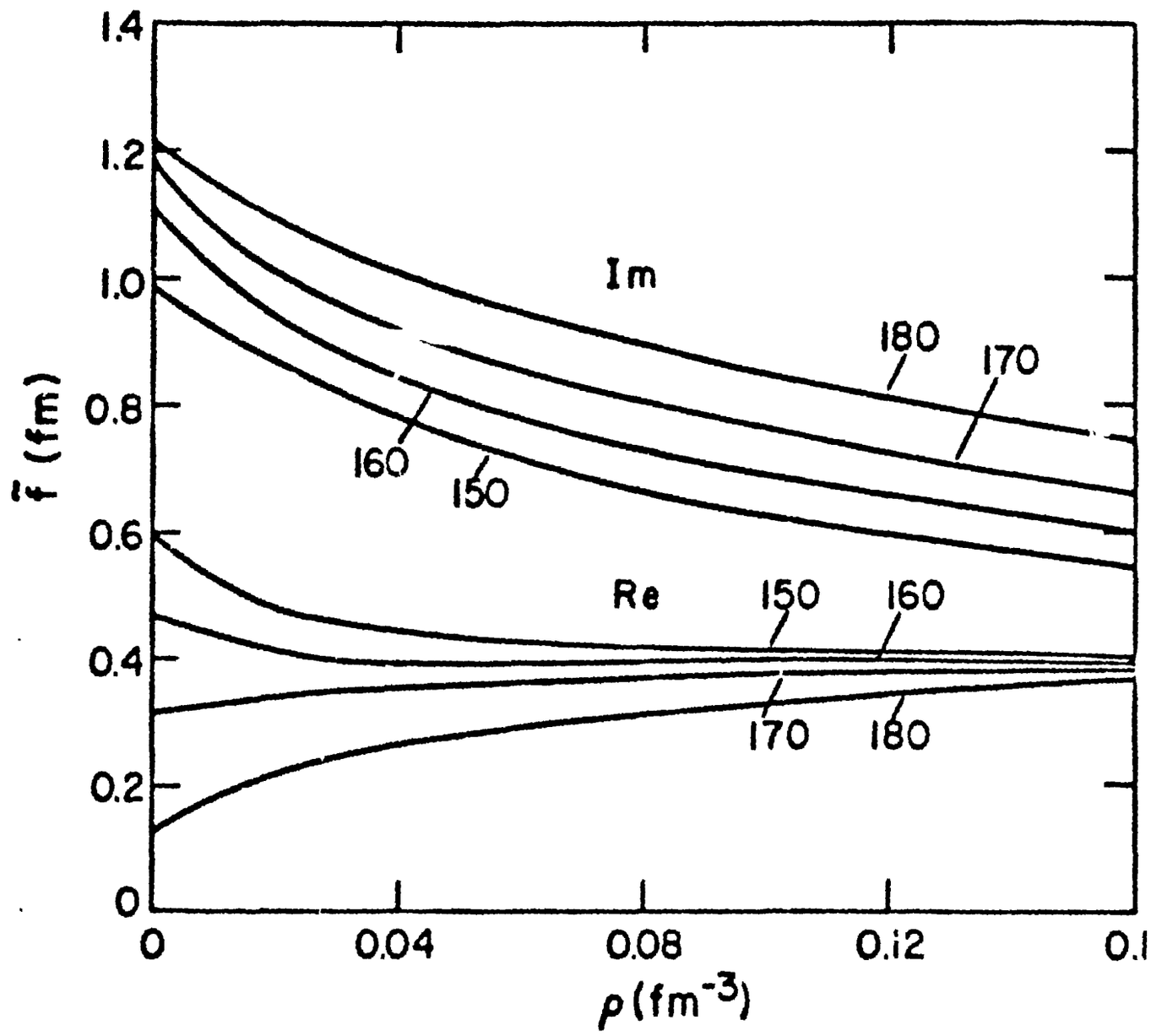


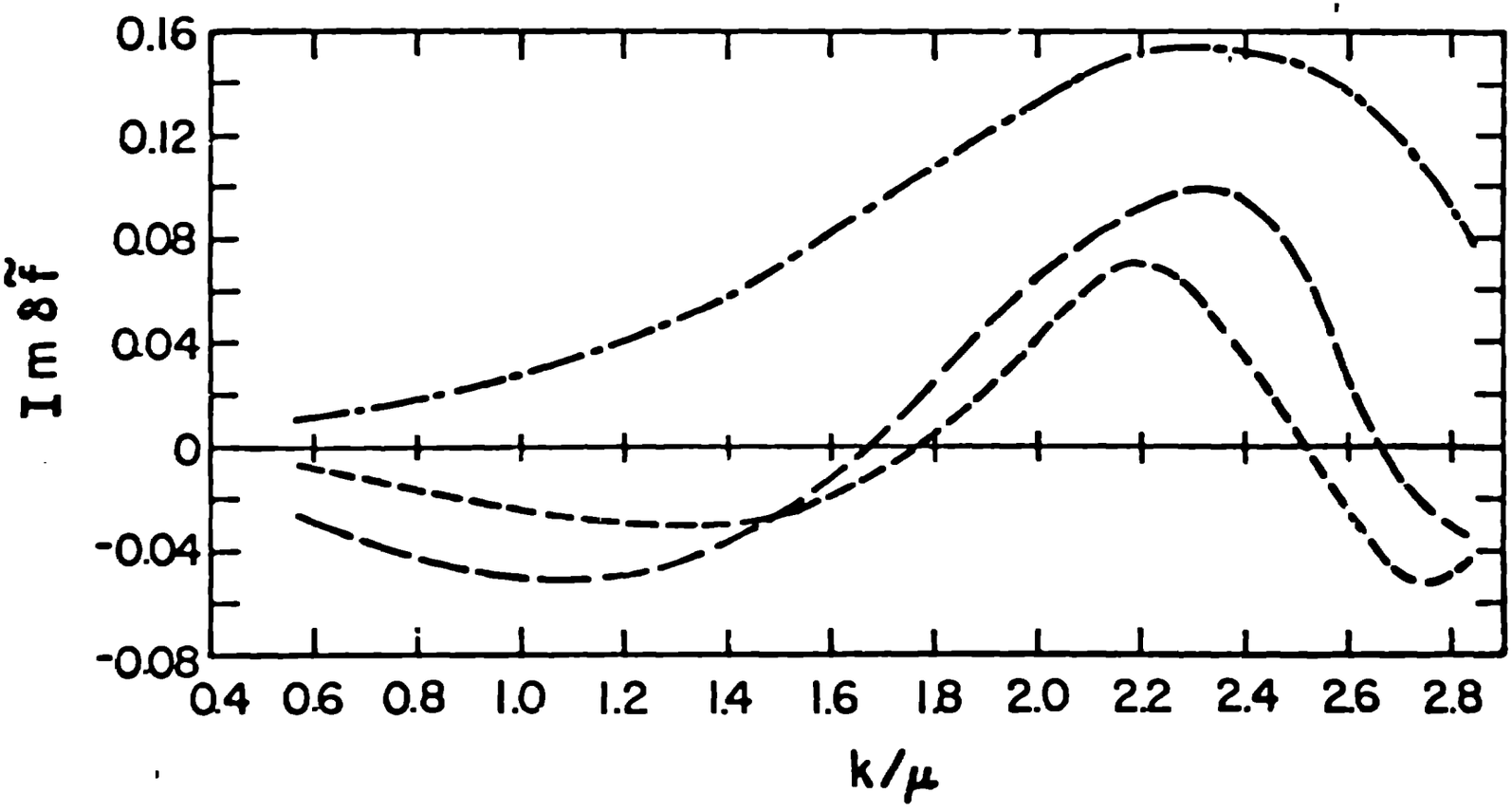


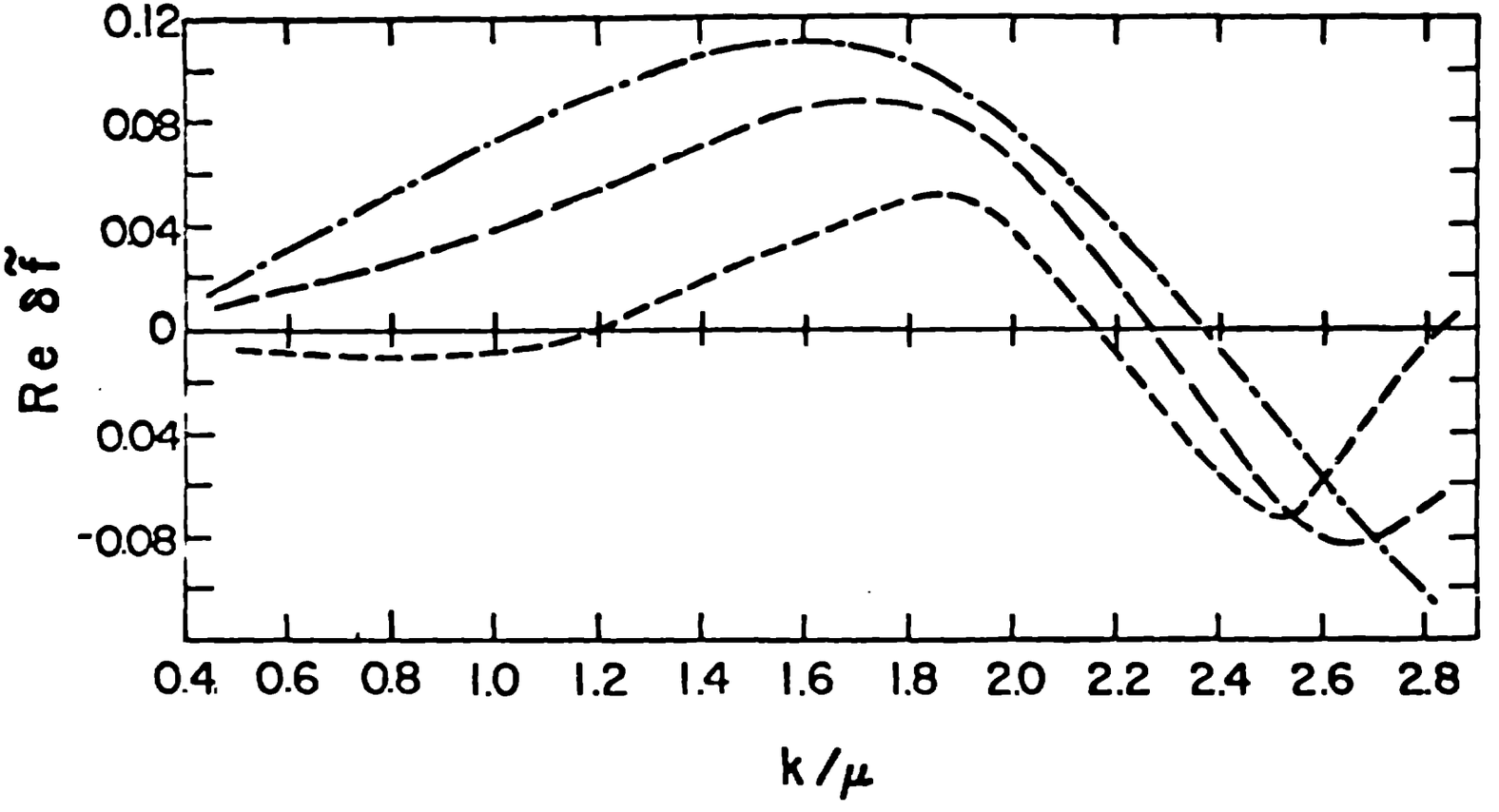


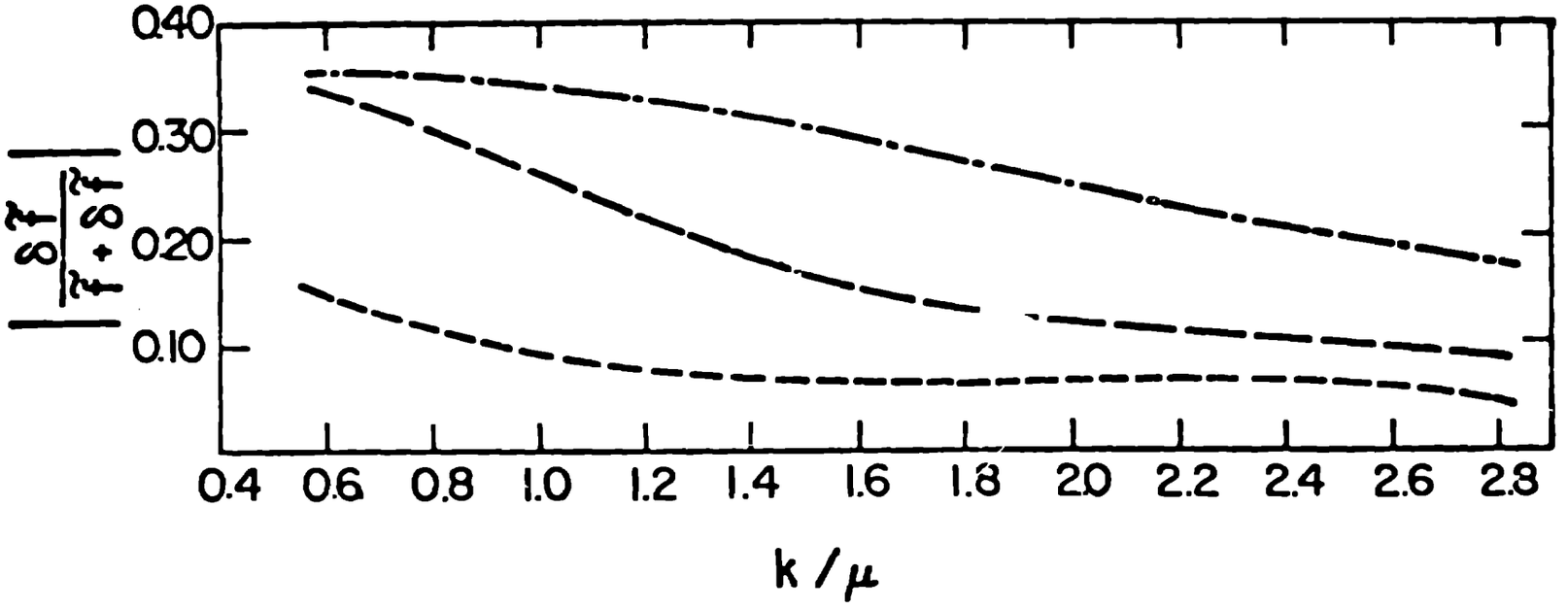

1.2
1.0
0.8
0.6
0.4
0.2
0
Im f̃
0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8
k/μ

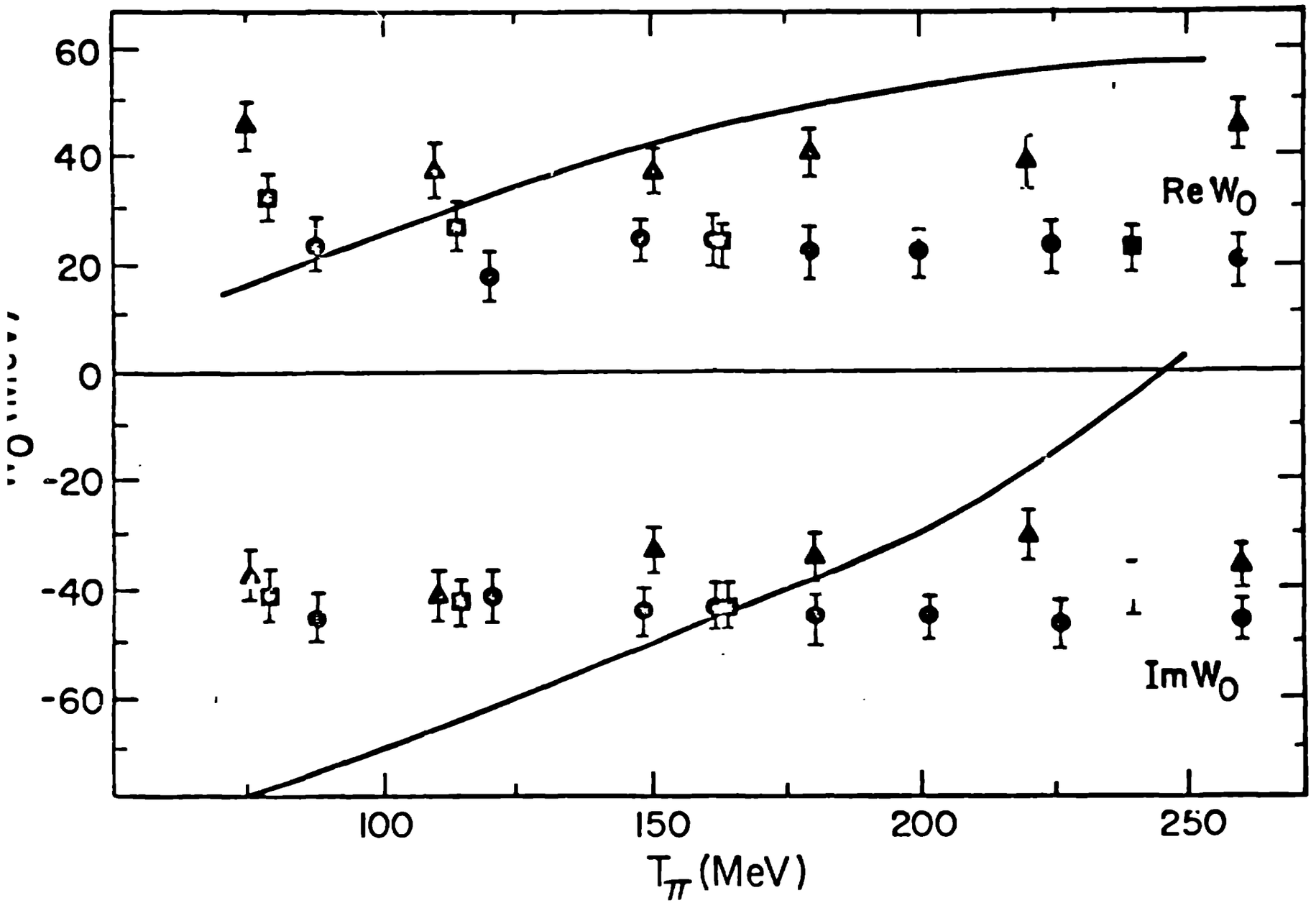

0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
Re f̃
0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8
k / μ

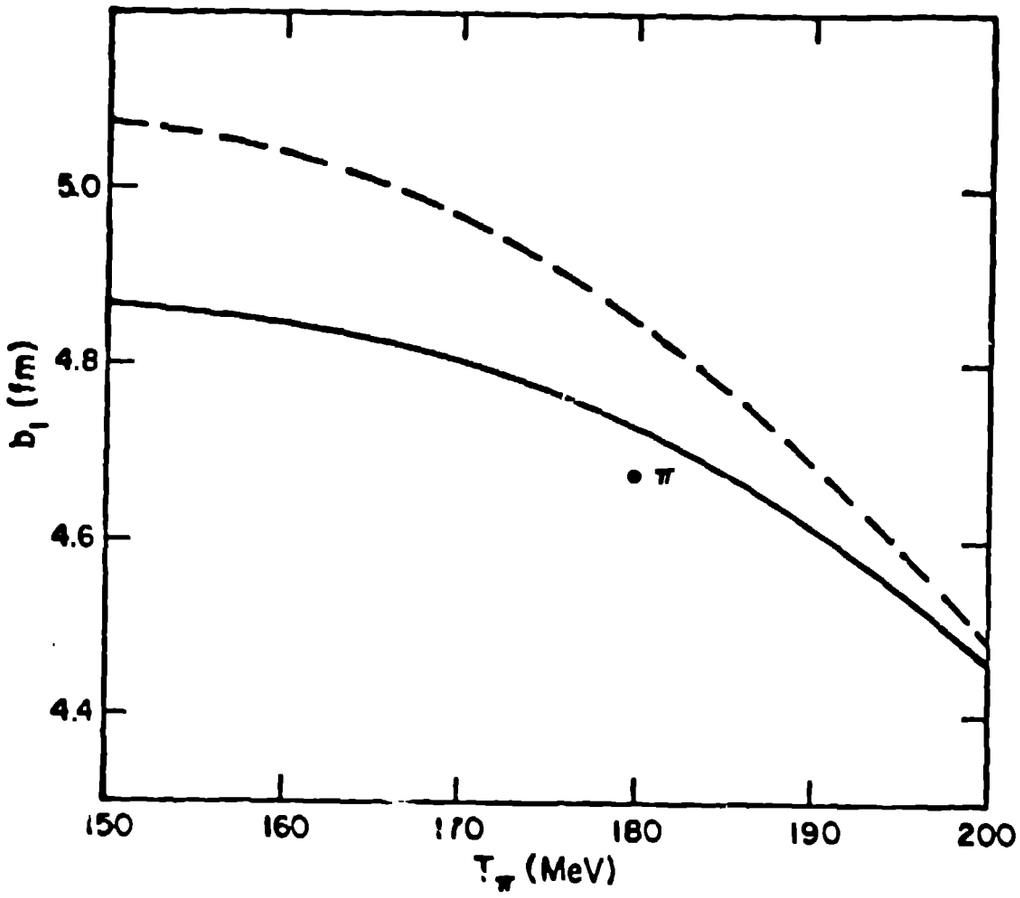

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
f (fm)
Im
180
170
160
150
Re
150
160
170
180
0
0.04
0.08
0.12
0.16
ρ (fm⁻³)

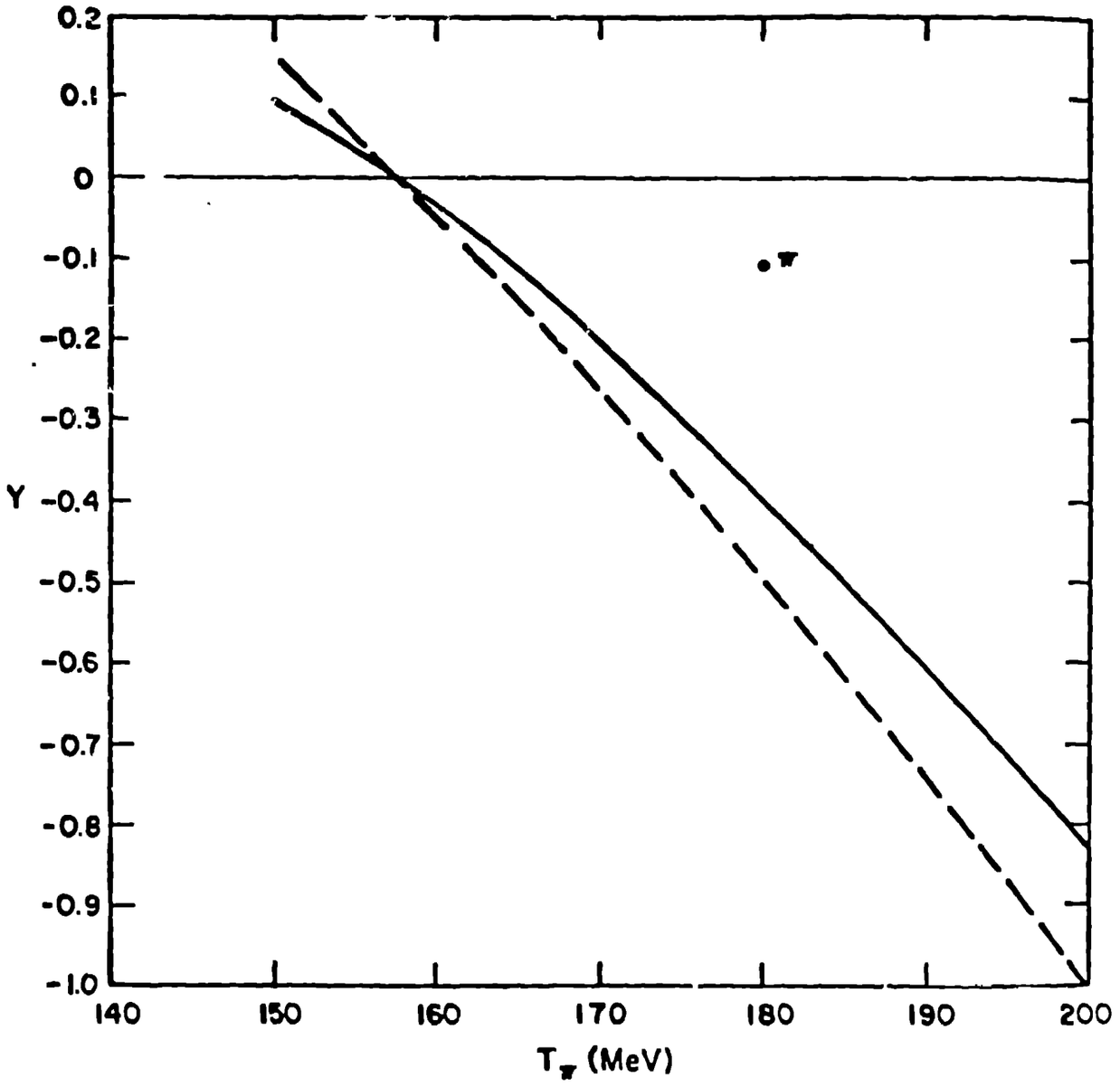

0.16
0.12
0.08
0.04
0
-0.04
-0.08
Im δ̃f
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
k/μ

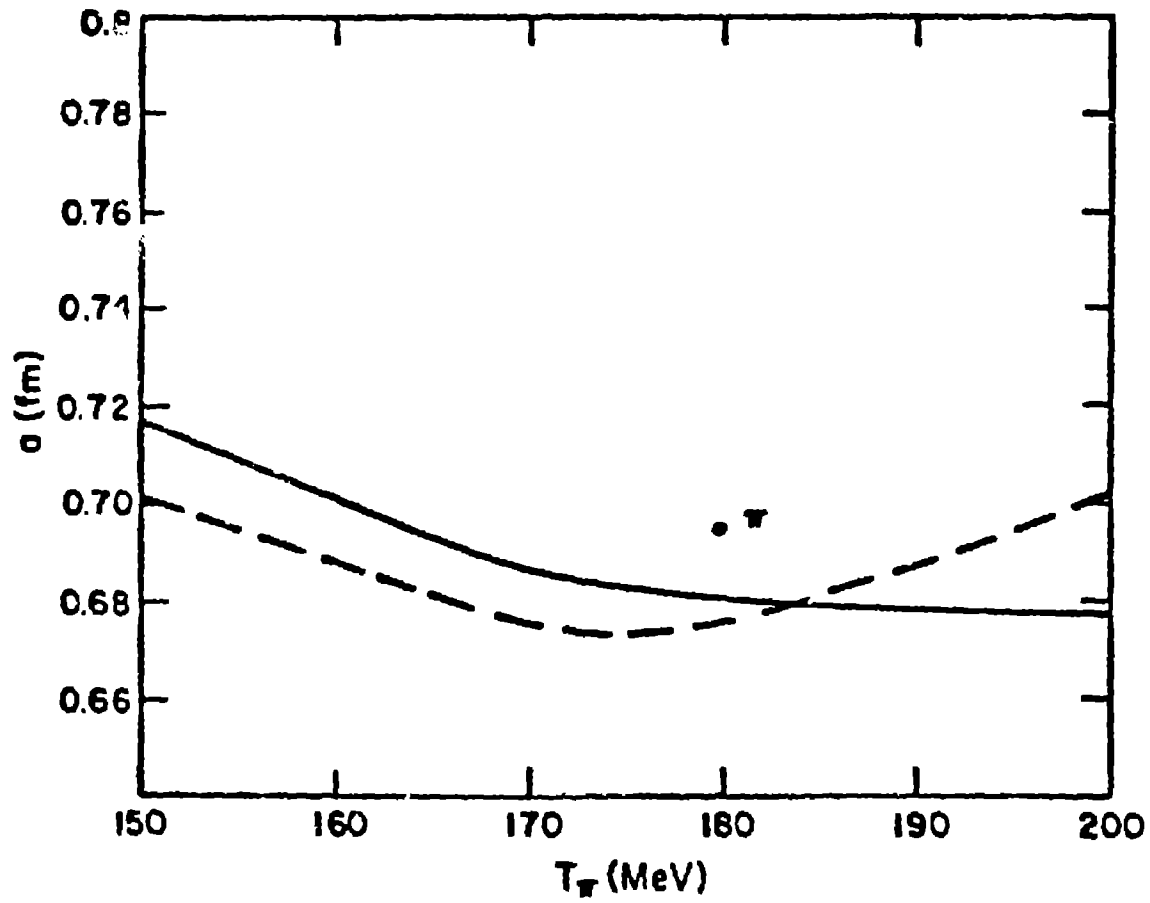

0.12
0.08
0.04
0
0.04
-0.08
Re δf̃
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
k/μ


0.40
0.30
0.20
0.10
$\frac{\sigma_T}{T + \sigma_T}$
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
k / μ


60
40
20
0
-20
-40
-60
Re W_0
Im W_0
100
150
200
250
T_π (MeV)


5.0
4.8
4.6
4.4
b_1 (fm)
π
150
160
170
180
190
200
T_π (MeV)


0.2
0.1
0
-0.1
-0.2
-0.3
Y -0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
140
150
160
170
180
190
200
T_π (MeV)


0.8
0.78
0.76
0.74
0.72
0.70
0.68
0.66
σ (fm)
150
160
170
180
190
200
T_π (MeV)